Quadratic programs

Problem statement

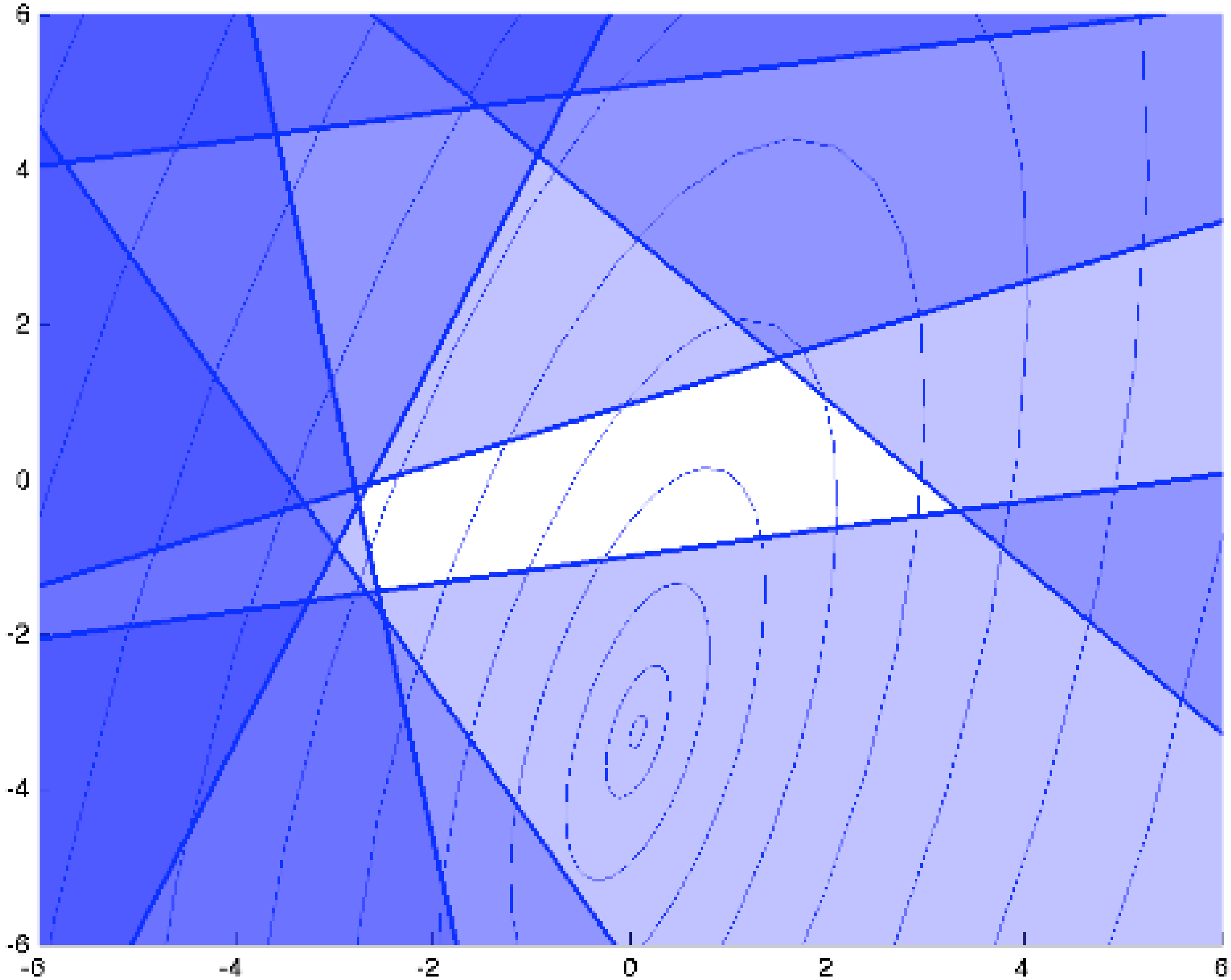
- Linear = ≥ ≤ constraints
- Quadratic objective

- minimize subject to

  $Ax + b \geq 0$

  $Ex + d = 0$

For example



QCQP

- Quadratic constraints make problem harder -- no longer called a QP!
- Quadratically-constrained QP instead

minimize $a^2 + b^2$ subject to

$a \geq x^2$, $b \geq y^2$

$3x + y = 6$

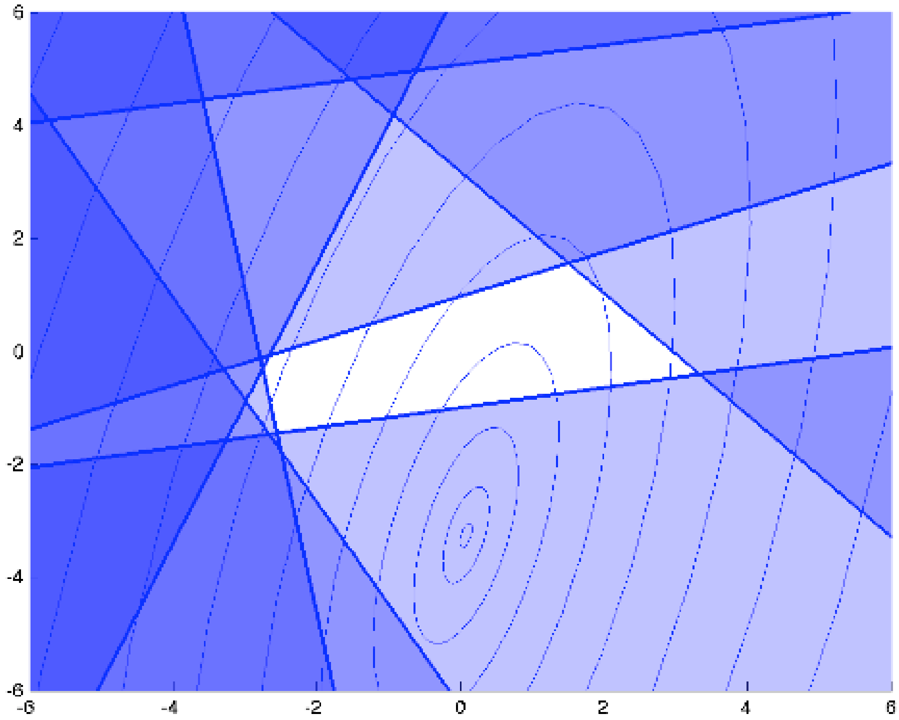
How hard are QPs?

- Max cut problem (NP-complete)

- Max cut QP

- QP is NP-hard (in fact, NP-complete)

Convex QPs



- +ve semidefinite objective
  - minimize x'Hx/2 + c'x
- Or -ve semidefinite for maximization
- Convex QP is about as hard as LP (poly-time, but very flexible)

Sketching convex QPs

- min $(x-4)^2 + (y-2)^2$ s.t.
- $3x + 2y \leq 6$
- $x \geq 0, \; y \geq 0$

Sketching ellipses

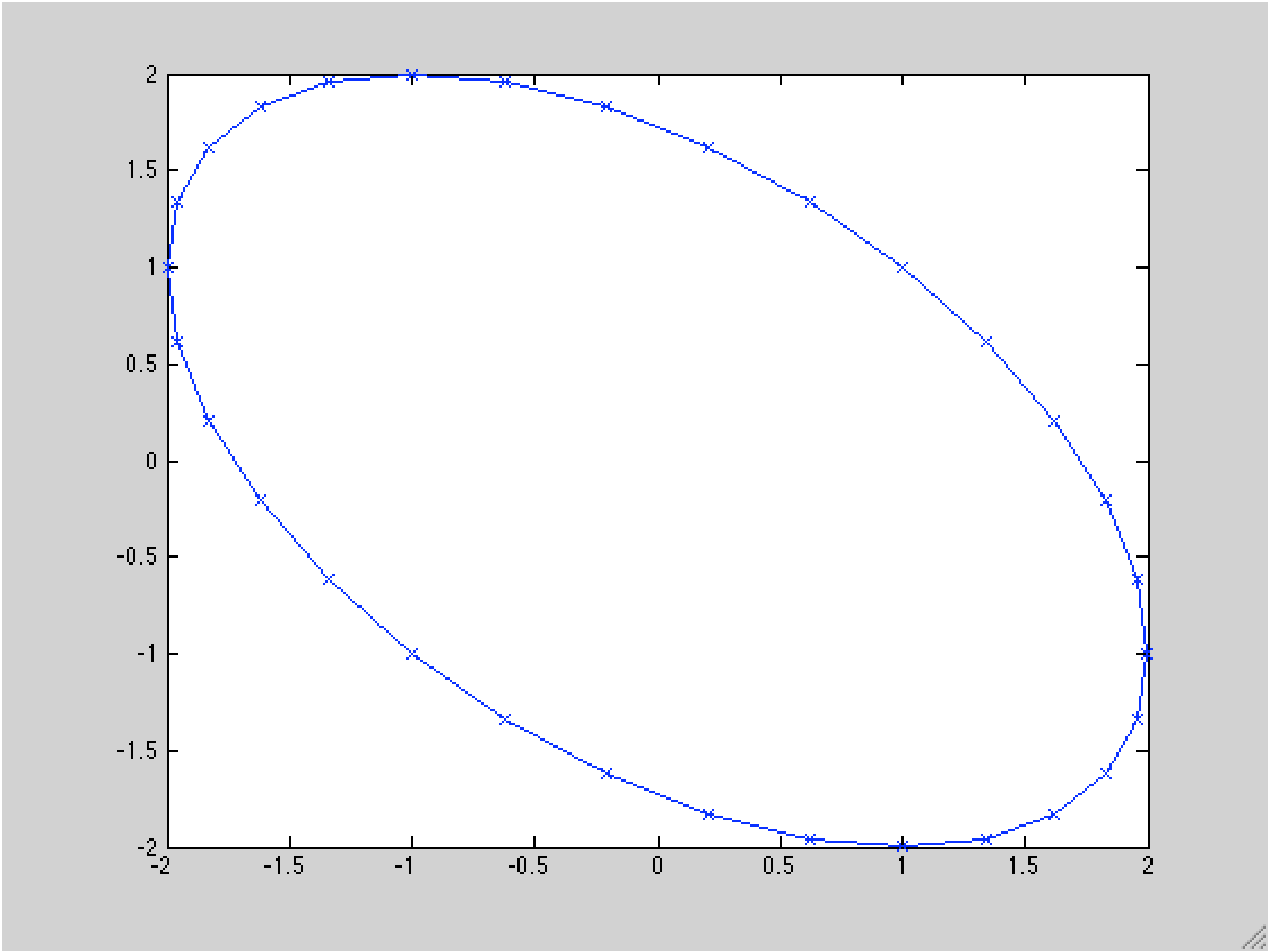
- min x'Hx/2 - c'x s.t. …
- If H = identity, circles centered @ c

- For positive-definite H = A'A:
- min x'A'Ax/2 - c'x   y = Ax

- In y-space, circles around $A^{-1}c$
- In x-space: images of circles under A

Sketching example

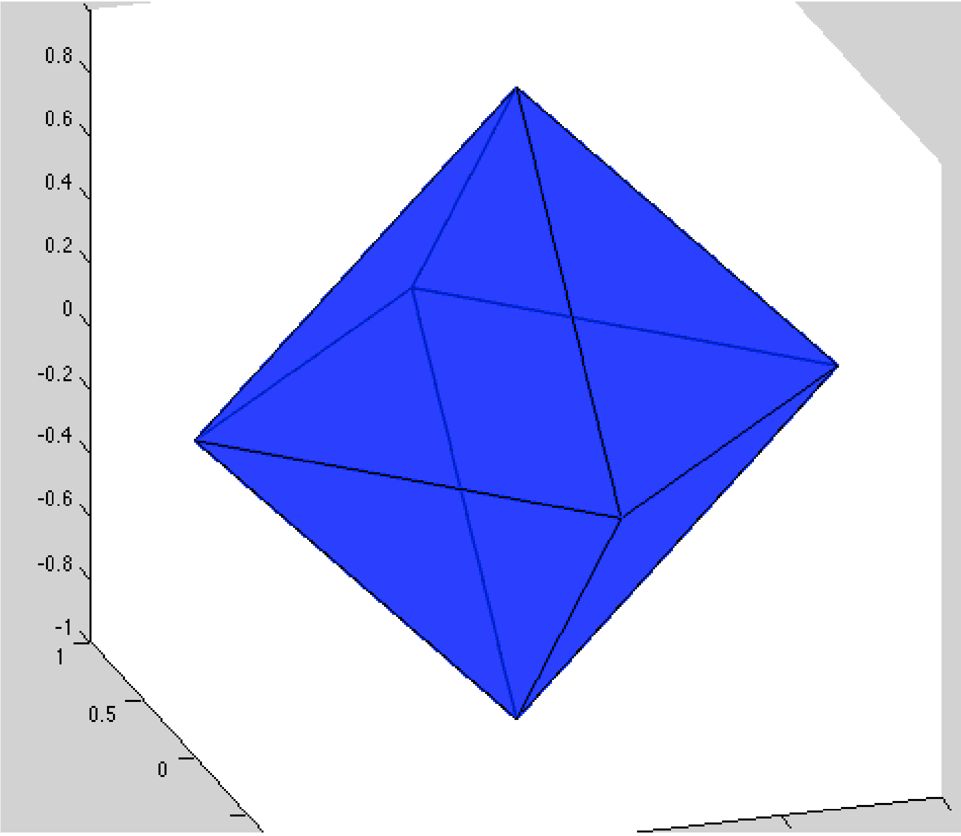
- $H = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix} = L\,L'$
- $L = \begin{pmatrix} 2 & 0 \\ -1 & \sqrt{3} \end{pmatrix}$


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

Quadratic program examples

Euclidean projection

- Find point closest to (3, 3, 3) in octahedron



Robust (Huber) regression

- Given points (x_i, y_i)
- L_2 regression:

  $\min_w \Sigma_i (y_i - x_i'w)^2$
- Problem:
- Solution: Huber loss

  $\min_w \Sigma_i Hu(y_i - x_i'w)$

  $Hu(z) =$

Huber loss as QP

- Hu(z):

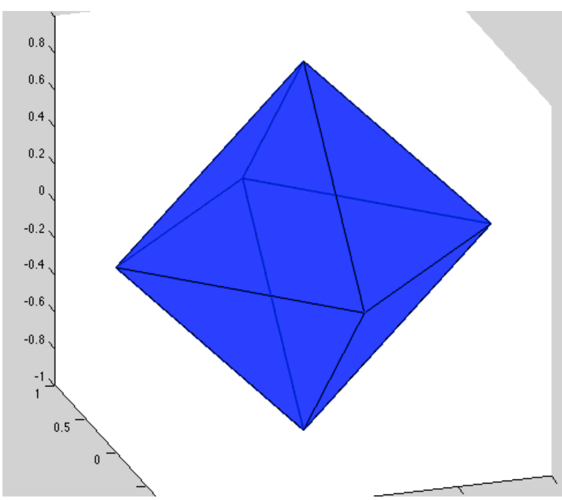
LASSO

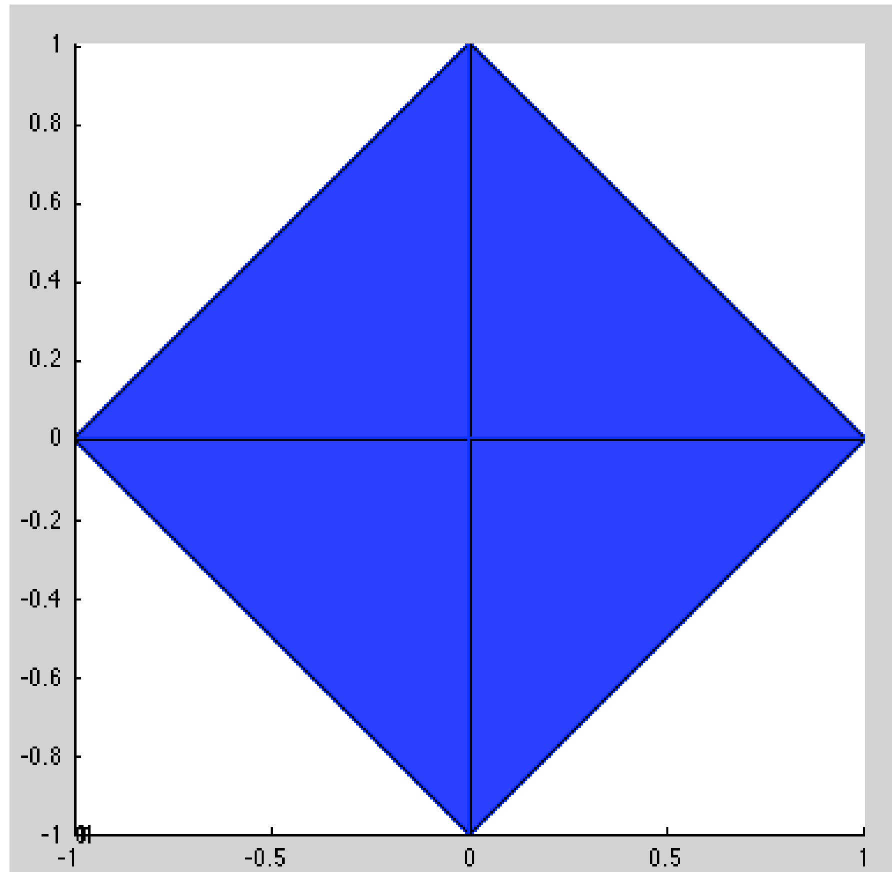
- "Least Absolute Shrinkage & Selection Operator"
- Problem: regression with many more features than examples

  $\min_w \Sigma_i (y_i - x_i'w)^2$
- Problem:

Why would LASSO work?

- Imagine constraining $||w||_1$
- Pretend quadratic is near-spherical





LASSO as QP

- Just like absolute value LP

$\min_w \Sigma_i (y_i - x_i'w)^2 + \Sigma_j s_j$

$s_j \geq w_j$

$s_j \geq -w_j$

Support vector machines

Maximizing margin

- margin = y_i ($x_i \cdot \bar{w} - \bar{b}$)
- max s.t.

Administrative

- Submission directories should be present now. `/afs/andrew/course/10/725/Submit/`***your-ID*** Check yours!
  - e.g., submit a small file "test.txt"
- We don't want to hear about problems late on night before due date…

- Audit deadline is tomorrow (1/29)
  - make sure we know about your preference
  - no need to tell us again if you signed up on Wed.
  - we are trying to accomodate as many as possible