LP duality cheat sheet

min $c'x + d'y$ s.t.	max $p'v + q'w$ s.t.
$Ax + By \geq p$	$A'v + E'w = c$
$Ex + Fy = q$	$B'v + F'w \leq d$
x free, $y \geq 0$	$v \geq 0$, w free

Swap RHS and objective
Swap max/min

Transpose constraint matrix
+ve vars yield ≤, free vars yield =

Linear feasibility problem

min $c'x$ s.t.

$Ax + b \geq 0$

find x s.t.

$Ax + b \geq 0$

Separation oracle

Ellipsoid preview

Difficulties

- How do we get bounding sphere?
- How do we know when to stop?
- Bound region gets complicated–how do we find its center?

Bounding a partial ellipsoid

- General ellipsoid w/ center x_C, radius R:

- Halfspace: $p^T x + q \leq 0$
- Translate to origin, scale to be spherical

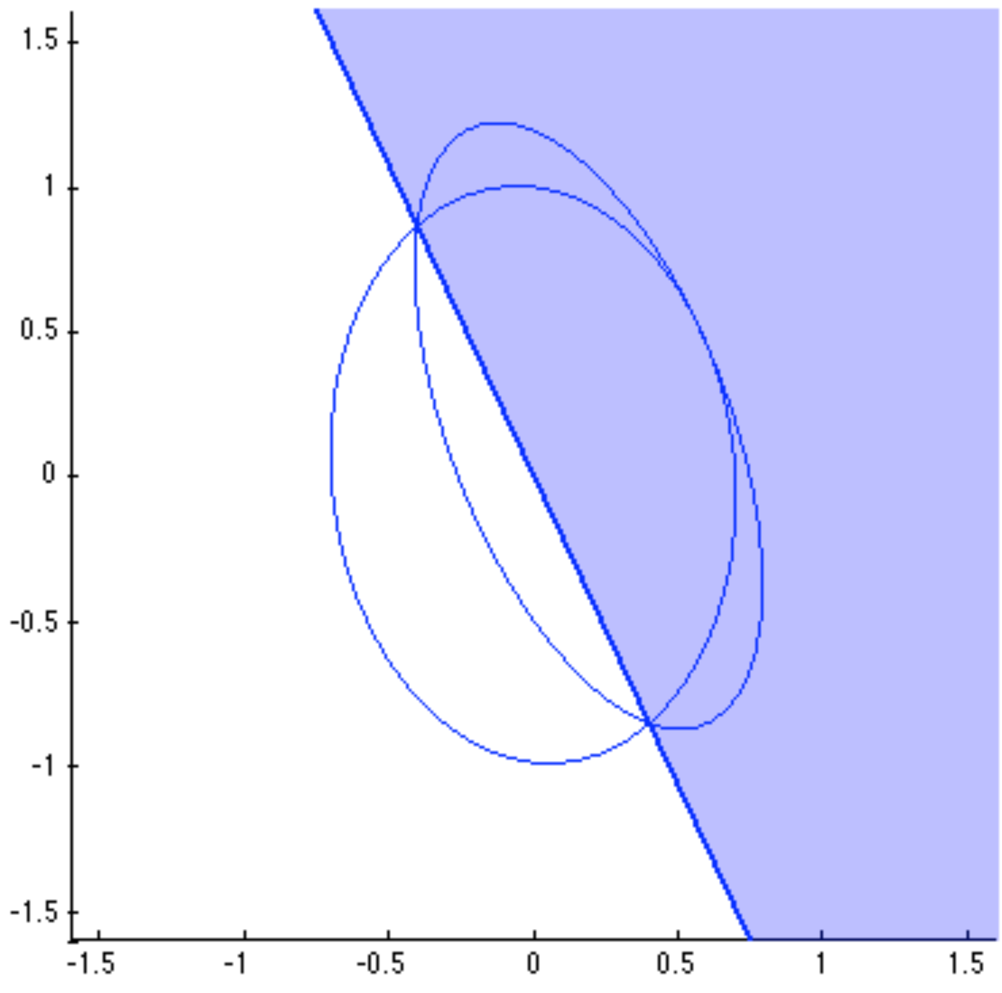
  y = x =

Bounding a partial sphere

- Rotate so hyperplane is axis-normal

- New center x_c:
- New shape A:

For example



Ellipsoid algorithm

- Want to find x s.t. $Ax+b+\eta \geq 0$
- Pick R s.t. $||x^*|| \leq R$
- $E_0 := \{ x \mid ||x|| \leq R \}$
- Repeat:
  - $x_t :=$ center of E_t
  - ask whether $Ax_t + b \geq 0$
    - yes: declare feasible!
    - no: get separating hyperplane
  - $E_{t+1} := \text{bound}(E_t \cap \{ x \mid p_t^T x \leq p_t^T x_t \})$
  - if $\text{vol}(E_{t+1}) \leq \varepsilon \text{vol}(E_0)$: declare infeasible!

Getting bounds

- How big do L, U need to be?
- How big does R need to be?
- What should η be?
- How small does ε need to be?

Other algorithms

- Ellipsoid is polynomial, but slow
- Some other algorithms:
  - simplex: exponential in worst case, but often fast in practice
  - randomized simplex: polynomial [Kelner & Spielman, 2006]
  - interior point: polynomial
  - subgradient descent: weakly polynomial, but really simple, and fast for some purposes

What’s a subgradient?

Subgradient descent for SVMs

- $\min_{s,w,b} \ ||w||^2 + C\sum_i s_i$ s.t.
  $y_i(x_i^T w - b) \geq 1 - s_i$
  $s_i \geq 0$
- Equivalently,

Subgradient in SVM

- $\min_w L(w) = ||w||^2 + C \sum_i h(y_i x_i^T w)$
- Subgradient of h(z):

- Subgradient of L(w) wrt w: