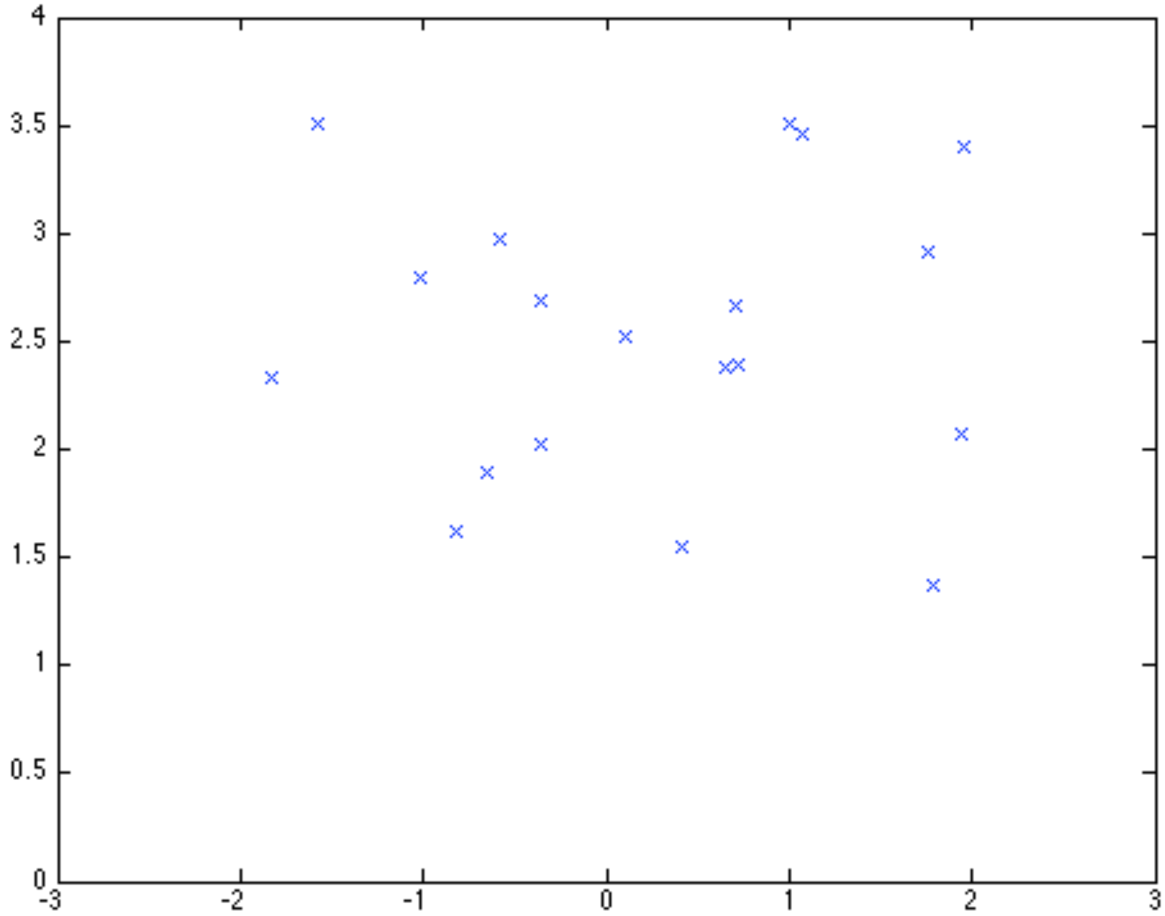
Positive definite max-QP

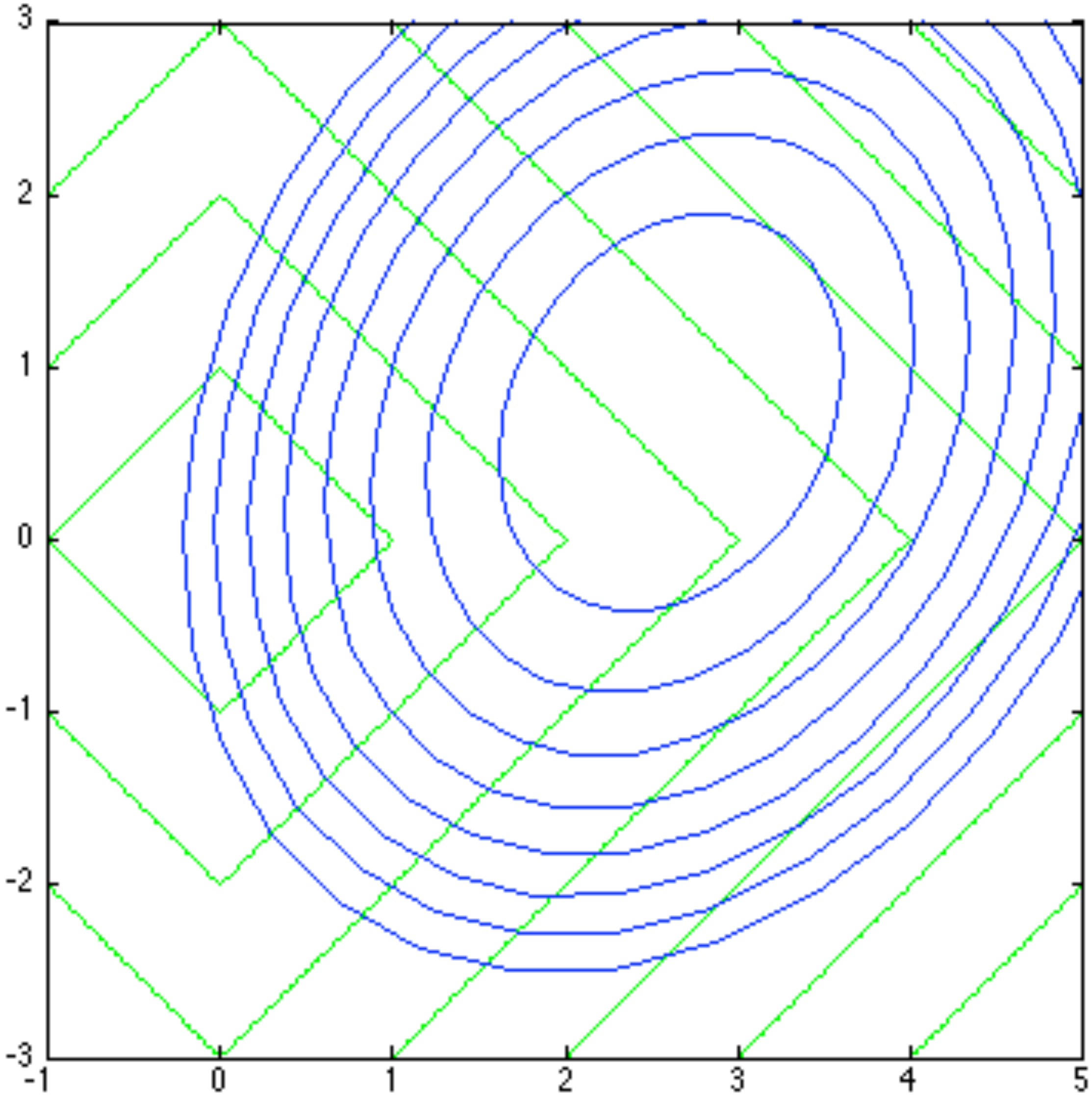
- Recall max-cut
- max $v'E(1-v)$ s.t. $0 \le v \le 1$

- max $v'(E + kI)(1-v)$ s.t. $0 \le v \le 1$

LASSO revisited



LASSO objective terms



Support vector machines

Maximizing margin

- margin = $y_i \, (x_i \cdot \bar{w} - \bar{b})$
- max s.t.

Administrative

- Submission directories should be present now. `/afs/andrew/course/10/725/Submit/`***your-ID***
  Check yours!
  – e.g., submit a small file “test.txt”
- We don’t want to hear about problems late on night before due date…
- Registration deadline: yesterday
  – everyone’s in!
  – to audit: register + get signed form + turn in to HUB
  – I still have one signed form

Duality

What if we're lazy

- A "hard" LP:

  min $x + y$ s.t.

  $x + y \geq 2$

  $x, y \geq 0$

OK, we got lucky

- What if it were:

$$\min \; x + 3y \quad \text{s.t.}$$

$$x + y \geq 2$$

$$x, y \geq 0$$

How general is this?

- What if it were:

  min $px + qy$ s.t.

  $x + y \geq 2$

  $x, y \geq 0$

Let’s do it again

- Note ≤ constraint

  min $x - 2y$ s.t.

  $x + y \geq 2$

  $x, y \geq 0$

  $x, y \leq 3$

And again

- Note = constraint

  min $x - 2y$ s.t.

  $x + y \geq 2$

  $x, y \geq 0$

  $3x + y = 2$

Summary of LP duality

- Use multipliers to write combined constraints
  - $\geq$
  - $\leq$
  - $=$
- Constrain multipliers to give us a bound on objective
- Optimize to get tightest bound

The Lagrangian

min $x + y$ s.t.

$x + y \geq 2$

$x, y \geq 0$

- $L(a,b,c,x,y) =$

  $[x + y] - [a\,(x + y - 2) + bx + cy]$

- $\min_{x,y} \max_{a,b,c \geq 0} L(a,b,c,x,y)$

Lagrangian cont'd

min $x + y$ s.t.
$x + y \geq 2$
$x, y \geq 0$

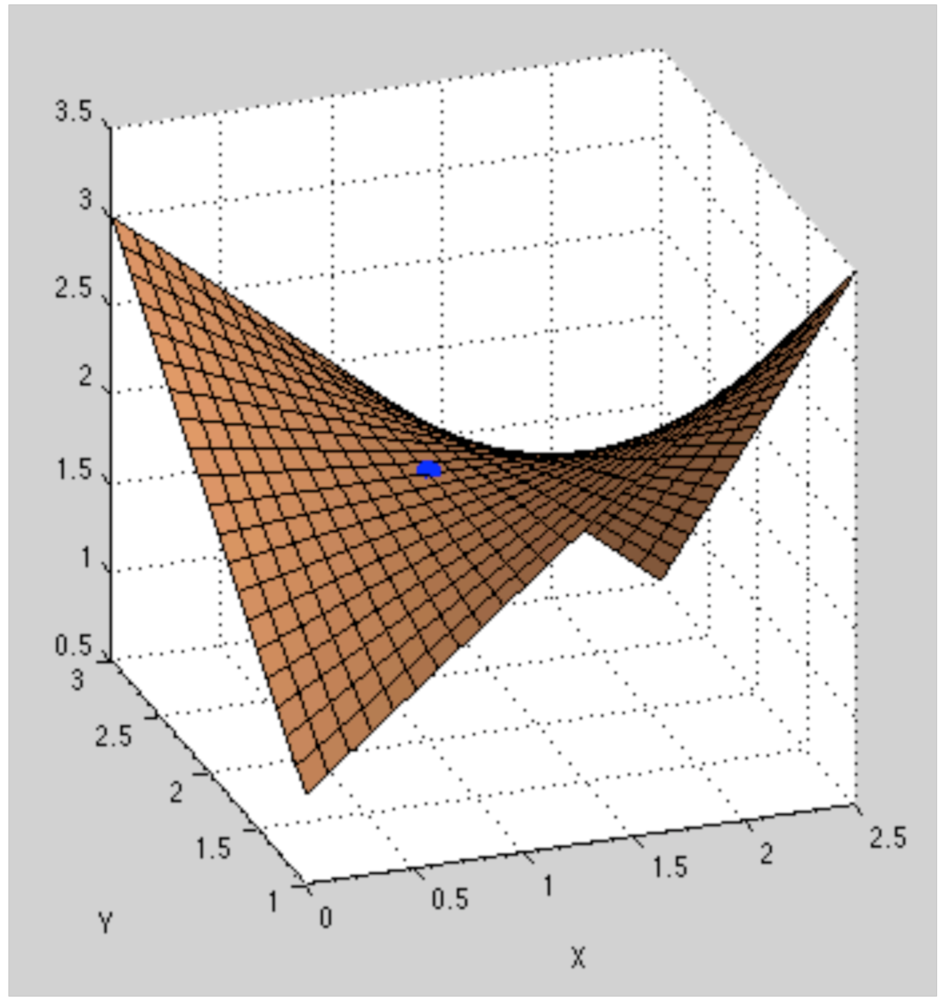
- $L(a,b,c,x,y) =$

  $[x + y] - [a\,(x + y - 2) + bx + cy]$

- $\min_{x,y} \max_{a,b,c \geq 0} L(a,b,c,x,y)$

Saddle-point picture

- min y s.t. y ≥ 2



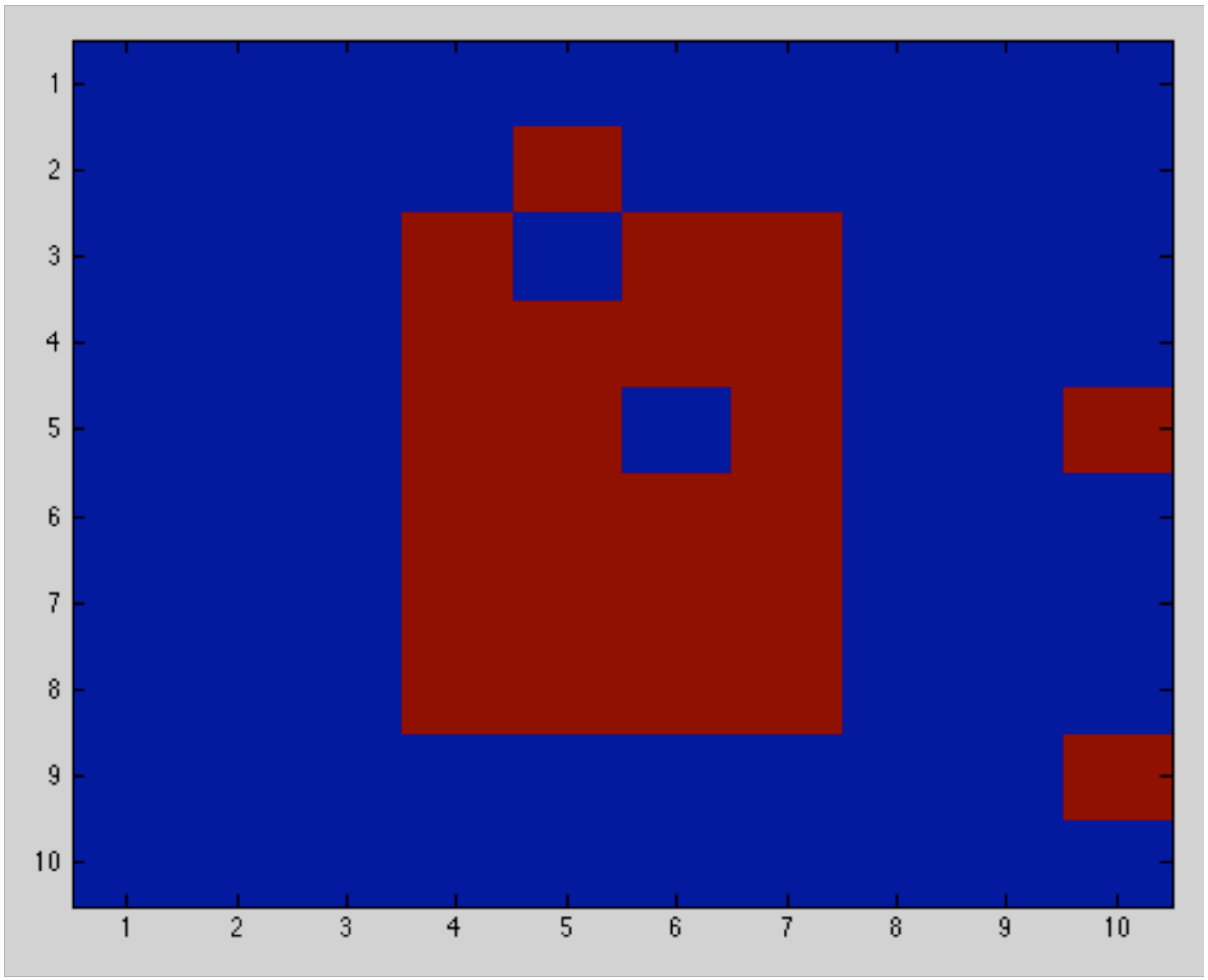
Example: max flow

- Given a directed graph
  - edges $(i,j) \in E$
  - flows f_{ij}, capacities c_{ij}
  - source s, terminal t ($c_{ts} = \infty$)
- max f_{ts} s.t.
  - positive flow
  - capacity
  - flow conservation

Dual of max flow

Interpreting dual

min cut: image segmentation



What about QP duality?

- min $x^2 + y^2$ s.t.

  $x + 2y \geq 2$

  $x, y \geq 0$
- How can we lower-bound OPT?

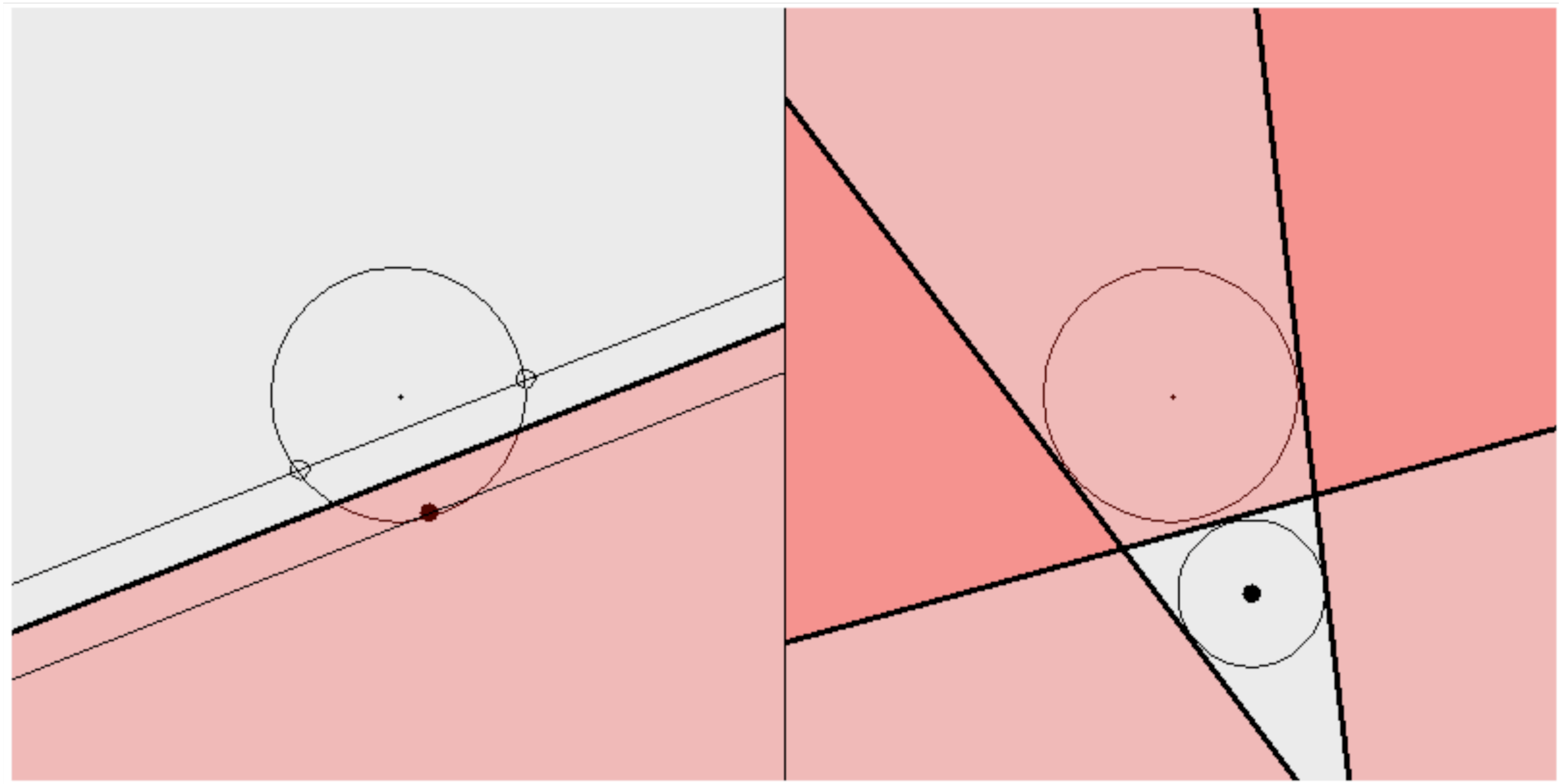
Works at other points too

- min $x^2 + y^2$ s.t.

  $x + 2y \geq 2$

  $x, y \geq 0$
- Try Taylor @ $(x, y) = (v, w)$

Example: SVMs



SVM duality

- Recall: min s.t.
- Taylor bound objective:
- Generic constraint:

- To get bound, need:

SVM dual

- $\max_{\alpha,v} \Sigma_i \alpha_i - ||v||^2/2$ s.t.

  $\Sigma_i \alpha_i y_i = 0$

  $\Sigma_i \alpha_i y_i x_{ij} = v_j$ for all j

  $\alpha_i \geq 0$ for all i

Perpendicular bisector