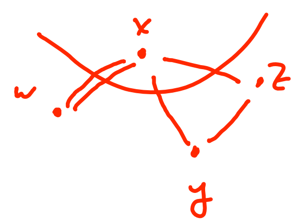
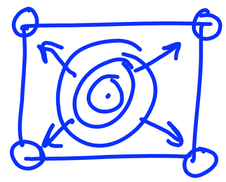
Positive definite max-QP

- Recall max-cut
- max v'E(1-v) s.t. 0 ≤ v ≤ 1

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$



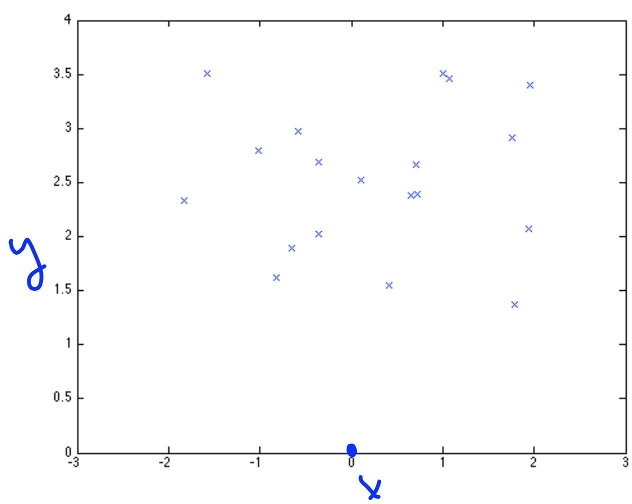
- max v'E(1-v) + k(v'v–1'v) s.t. 0 ≤ v ≤ 1
  - extra term quadratic
  - pushes away from (.5,.5,...,.5)
  - doesn't change relative order of corner points



w/ large k, overall objective is max +ve definite

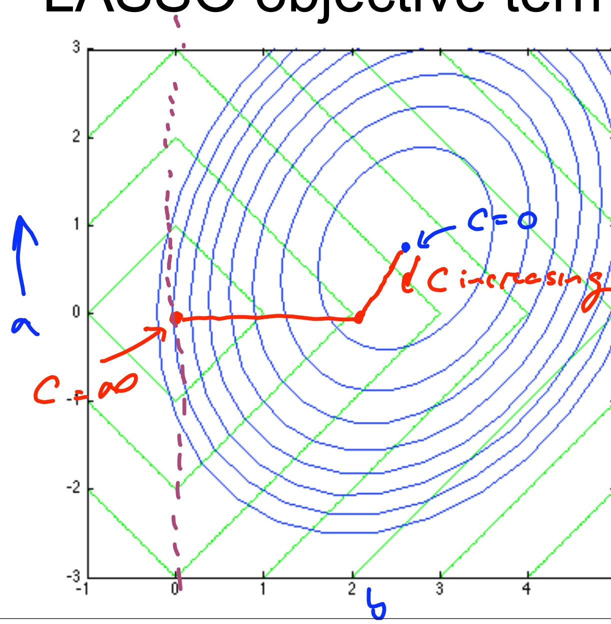
$$\min \sum_i (y_i - (a x_i + b))^2 + C(|a| + |b|)$$

LASSO revisited



$ax + b$

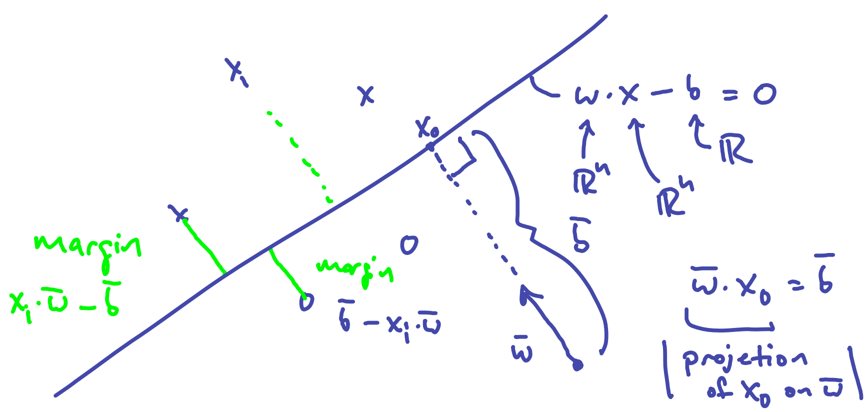
LASSO objective terms

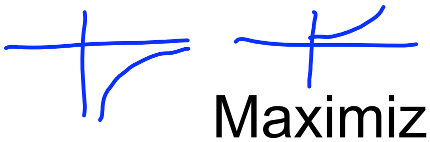


$\bar{w} = w / \|w\|$

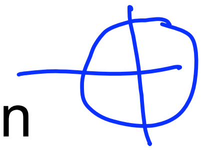
$\bar{b} = b / \|w\|$

Support vector machines





Maximizing margin



- margin = $y_i \, (x_i \cdot \bar{w} - \bar{b})$ $\quad \forall i$
- $\max\limits_{w \; b} \quad M \qquad$ s.t. $\; y_i(x_i \cdot \bar{w} - \bar{b}) \geq M$

$$\forall i \quad y_i \left(x_i \cdot \overbrace{\frac{w}{\|w\| M}}^{v} - \overbrace{\frac{b}{\|w\| M}}^{d} \right) \geq 1$$

$$\Rightarrow v = \frac{w}{M \|w\|} \qquad d = \frac{b}{M \|w\|} \qquad \|v\| = \frac{\|w\|}{M \|w\|} = \frac{1}{M}$$

$$\max_{v \; d} \; 1/\|v\| \quad \text{s.t.} \quad y_i(x_i v - d) \geq 1$$

$$\min_{v \; d} \; -1/\|v\| \Leftrightarrow \|v\|^2/2 \qquad \boxed{QP!}$$

ssh gg25@unix.andrew

scp

Administrative

- Submission directories should be present now. `/afs/andrew/course/10/725/Submit/`***your-ID*** Check yours!
  - e.g., submit a small file “test.txt”
- We don’t want to hear about problems late on night before due date…
- Registration deadline: yesterday
  - everyone’s in!
  - to audit: register + get signed form + turn in to HUB
  - I still have one signed form

Duality

What if we're lazy

- A "hard" LP:

  min $x + y$ s.t.

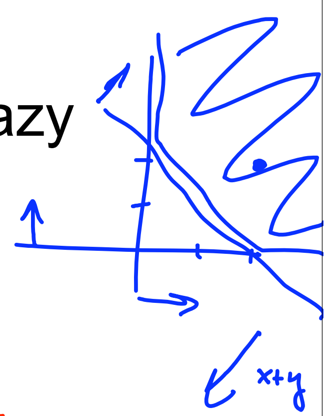
  $x + y \geq 2$

  $x, y \geq 0$

$x, y = 2, 2$

value 4

OPT ≥ 2



OK, we got lucky

- What if it were:

  min $\underline{x + 3y}$ s.t.

  $x + y \geq 2$

  $x, y \geq 0$

$$\begin{array}{r} x+y \geq 2 \\ 2y \geq 0 \\ \hline OPT = \underline{x+3y} \geq 2 \end{array}$$

How general is this?

- What if it were:

  min $\underline{px} + \underline{qy}$ s.t.

  $x + y \geq 2$

  $x, y \geq 0$

$a(x+y-2) + b(x) + c(y) \geq 0$

$a, b, c \geq 0$

$(a+b)x + (a+c)y \geq 2a$

$\boxed{\begin{array}{l} a+b = p \\ a+c = q \end{array}}$

$OPT = px + qy = (a+b)x + (a+c)y \geq \underline{2a}$

max $2a$ s.t.

DUAL

Let's do it again

- Note ≤ constraint

  min x – $\underline{2}$y s.t.

  x + y ≥ 2

  x, y ≥ 0

  x, y ≤ 3 $\longrightarrow$ $-x \geq -3$, $-y \geq -3$, $d, e \leq 0$

$a(x+y-2) + bx + cy + d(x-3) + e(y-3) \geq 0$

$a, b, c \geq 0$

$\underbrace{(a+b+d)}_{=1}x + \underbrace{(a+c+e)}_{=-2}y + -2a - 3d - 3e \geq 0$

And again

- Note = constraint

  min x – $\underline{2}$y s.t.

  x + y ≥ 2

  x, y ≥ 0

  $\rightarrow$ 3x + y = 2

$a(x+y\underline{-2}) + bx + cy + d(3x+y\underline{-2}) \geq 0$

$a, b, c \geq 0$

d free

opt $\geq 2a + 2d$

DUAL

$$\boxed{\begin{array}{l} \max 2a + 2d \text{ s.t.} \\ a + b + 3d = 1 \\ a + c + d = -2 \end{array}}$$

Summary of LP duality

- Use multipliers (Lagrange multipliers) to write combined constraints

  ≥ +ve L. M.

  ≤ −ve L. M.

  = free L. M.

- Constrain multipliers to give us a bound on objective
- Optimize to get tightest bound

The Lagrangian

primal →

$\min\ x + y$ s.t.
$x + y \geq 2$
$x, y \geq 0$

dual →

max $2a$ st
$a+b = 1$
$a+c = 1$
$a, b, c \geq 0$

- $L(a,b,c,x,y) =$

$$[x + y] - [a (x + y - 2) + bx + cy]$$

- $\min_{x,y} \max_{a,b,c\geq 0} L(a,b,c,x,y)$

$d, e, f = -a, -b, -c$

$$\min_{x,y} \max_{d,e,f \leq 0} x+y + d(x+y-2) + ex + fy$$

$$2a - [x(a+b-1) + y(a+c-1)]$$

min x + y s.t.
x + y ≥ 2
x, y ≥ 0

Lagrangian cont'd

- L(a,b,c,x,y) =
  [x + y] – [a (x + y – 2) + bx + cy]
- $\min_{x,y} \max_{a,b,c\geq 0}$ L(a,b,c,x,y)

Saddle-point picture

- min y s.t. y ≥ 2

$L(y,a) = y - [a(y-2)]$
$= y - ay + 2a$