Solving convex programs

- Linear programs:
- General CP:
- Interesting special cases: QP, SOCP, SDP

Separation oracle: QPs

- min q(x) st Ax = b, x ≥ 0

Separation oracle: SOCPs

- SOC constraint: $||Ax + b|| \leq c'x + d$
- Given x_0 that fails:

Separation oracle: SDPs

- SDP constraint:

  $A = x_1 A_1 + x_2 A_2 + \ldots$

  $A \in S_+$

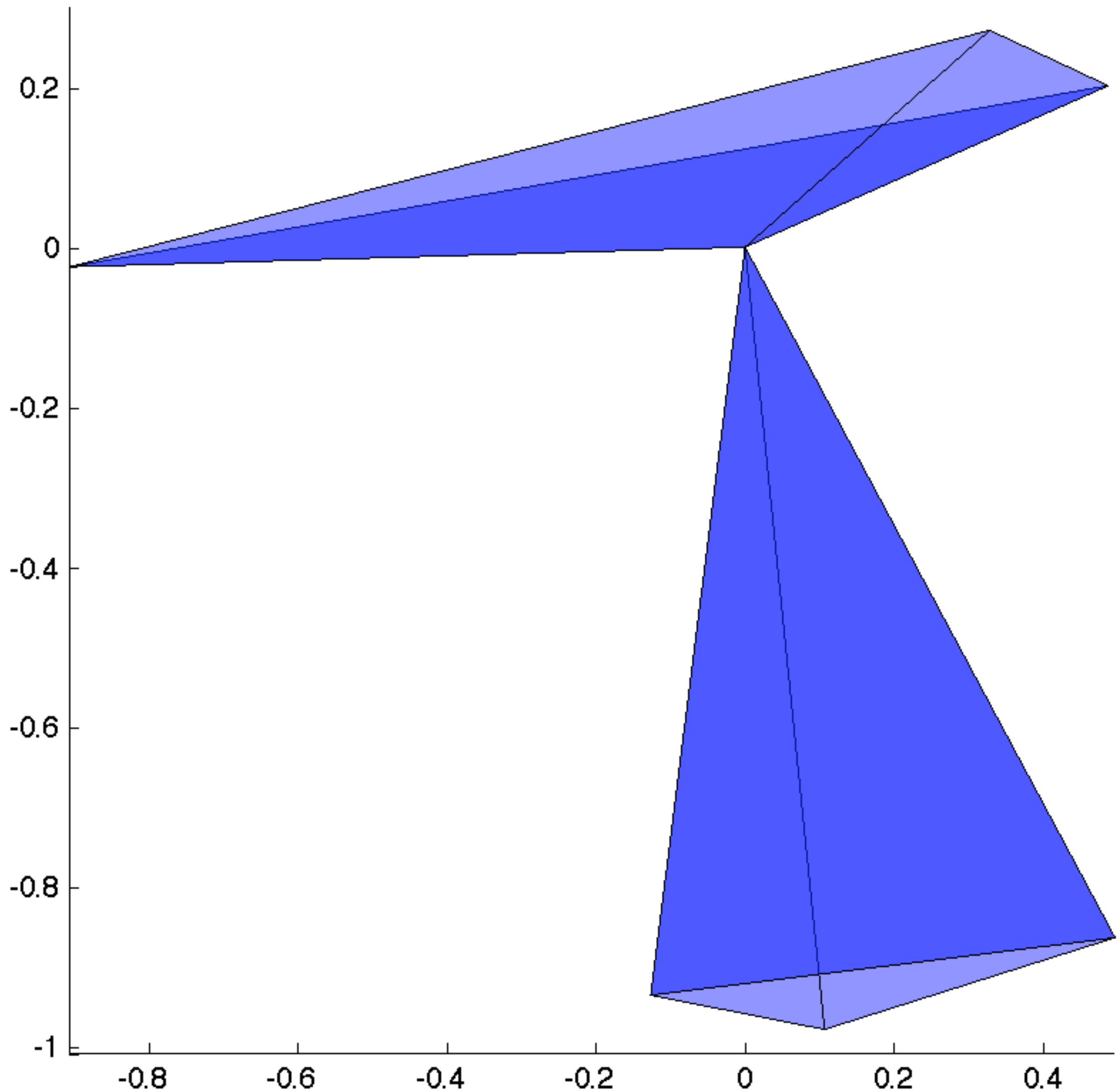
Convex duality

- Several new types of duality
  - convex cones
  - convex sets
  - convex functions
  - convex programs
- Generalize LP/QP duality
- Generalize norm duality (e.g., L_1 v. L_∞)

Cone duality

- Cone K (not necessarily convex)

- K* =


0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4

Examples of dual cones

- Halfspace $a^T x \geq 0$
- Subspace $\{ x \mid Ax = 0 \}$
- R_+^n
- SOC: $\{ (x, s) \mid \|x\|_2 \leq s \}$

- norm cone: $\{ (x, s) \mid \|x\| \leq s \}$

S_+ is self-dual

- S_+: $\{ A \mid A=A^T, x^TAx \geq 0 \text{ for all } x \}$

Ex: Euclidean distance matrices

- Given points x_i
- Matrix D: D_{ij} =

Properties of dual cones

- K^* is closed and convex

- $K^{**} = \text{cl conv } K$

- If K closed and convex,

Properties of dual cones

- $K_1 \quad K_2 \qquad K_2^* \quad K_1^*$

- $K_1 \quad K_2 \qquad K_2^* \quad K_1^*$
- If K_1, K_2 are closed and convex:

Intersection and union

- $(K_1 \cup K_2)^* =$

- $(K_3 \cap K_4)^* =$

Flat, pointed, solid, proper

- K is **flat** if:
- E.g., K =
- K is **pointed** if:
- E.g., K =
- K is **proper** if:
- E.g., K =

Generalized inequalities

- Given proper cone K
- $x \geq_K y$ iff $x - y \geq_K 0$ iff

- $x >_K y$ iff $x \geq_K y$ and x != y
- $x \leq_K y$ and $x <_K y$: as expected
- Transitive:
- Examples:

Application: multi-criterion optimization

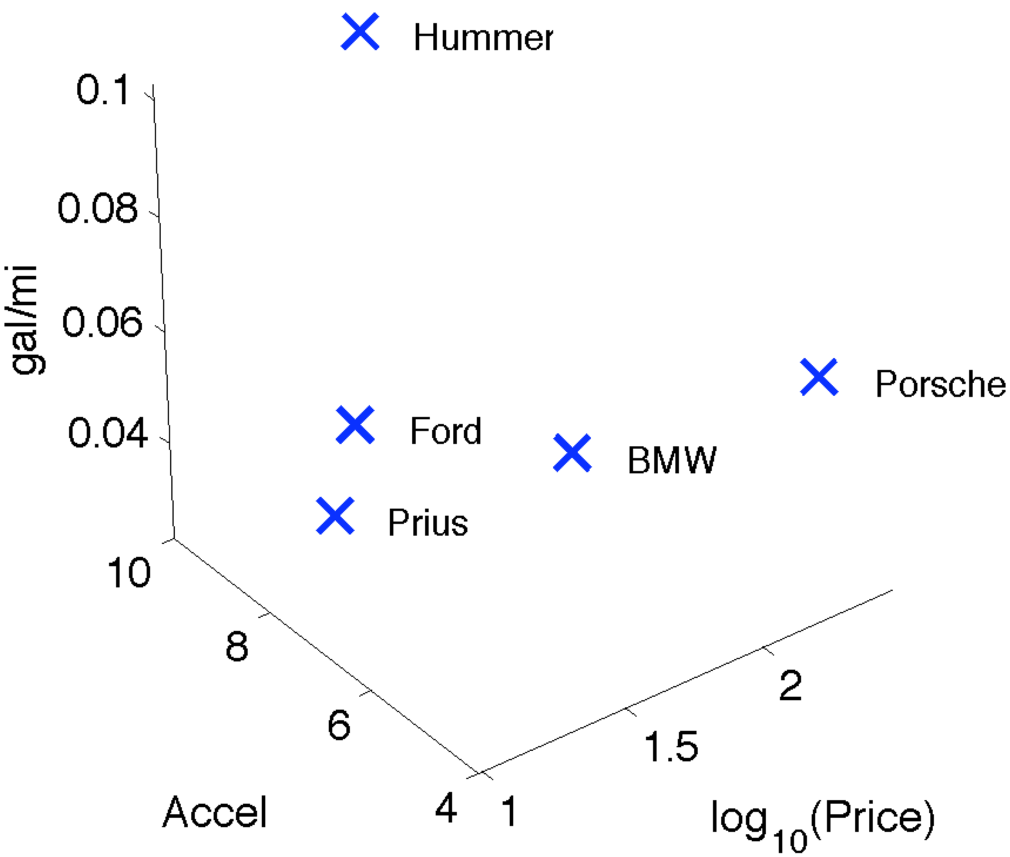
- Ordinary feasible region
- Indecisive optimizer: wants all of

Buying the perfect car

$K	0-60	MPG

Pareto optimality

x* Pareto optimal =



Pareto examples

Scalarization

- To find Pareto optima of convex problem:

Dual sets

- Any convex set C
  - e.g.,
- can be represented as intersection of
  - a convex cone:
  - and the hyperplane:
- Dual set: C* =

For example

- Dual of unit sphere

Equivalent definition

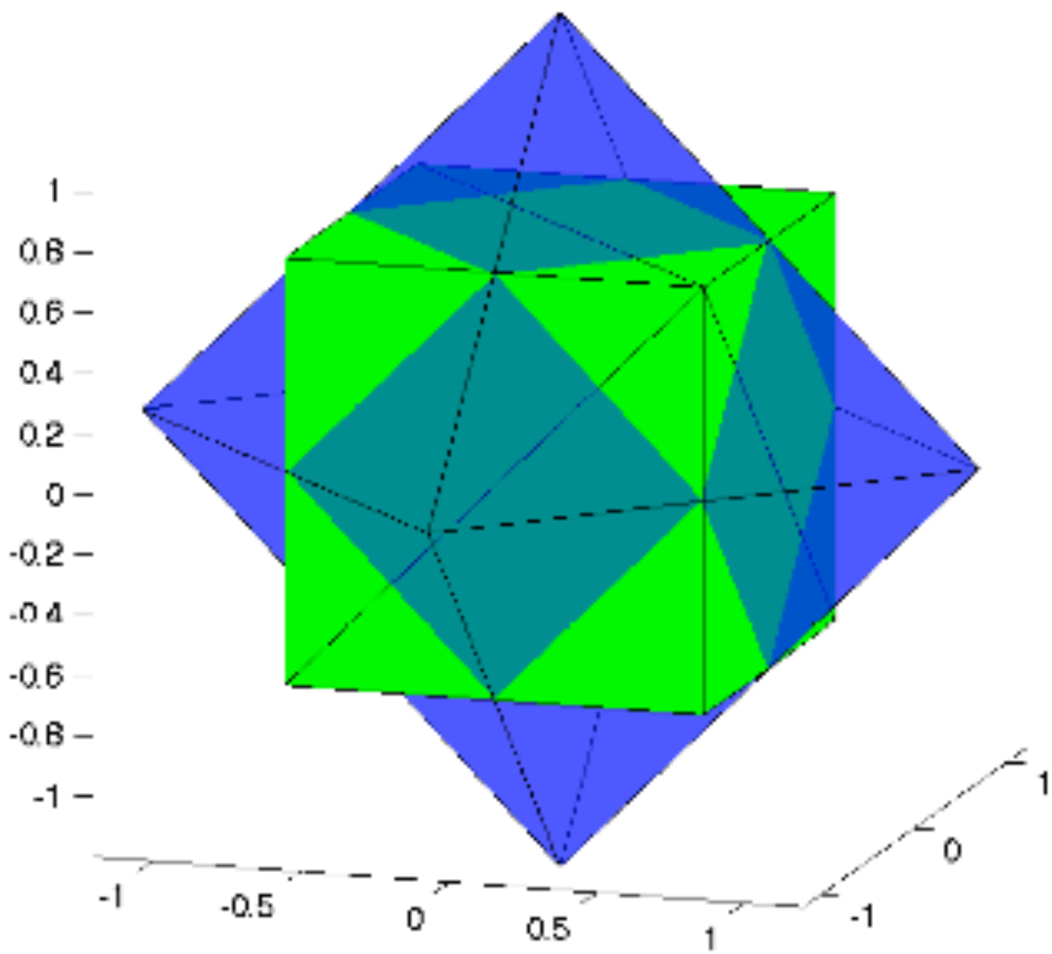
$C^* = \{ y \mid$

More examples

- $\{ x \mid x^T A x \leq 1 \}$ A invertible

- Unit square $\{ (x, y) \mid -1 \leq x,y \leq 1 \}$

Cuboctahedron



Dual-norm balls

- Dual norm definition

  $||y||_* = \max$

- $\{ x \mid ||x|| \leq 1 \}^* =$