Convex optimization

minimize subject to

e.g., min s.t.

Linear inequalities:
Positivity:

Terminology

- Feasible
- Infeasible
- Unbounded

- Two convex programs are equivalent if:

Transformations of CPs

- Optimizing some variables
- min $f(x,y) = y^2 + (x - 2y)^2 / 2$

Transformations of CPs

- Implicit constraints
- min x^2 s.t. $x \geq 1$
- min $x^2 + I_{x\geq 1}(x)$
  $I_{x\geq 1}(x) =$
- For any set S, $I_S(x) =$

Transformations of CPs

- Epigraph form:
- min x^2 s.t. $x \geq 2$
- min t s.t.

Optimality

- Optimal value:
  $v^* = \inf \{ f(x) \mid g_i(x) \leq 0 \ (\forall i), Ax = b \}$
- Definition of $v^* = \inf S$:
- For example: $\inf \{ e^x \mid x \text{ real} \} =$

Optimal point

- x^* is optimal iff:

- Is there always an optimal point?
  - ex. 1:

  - ex. 2:

Local optima

- min f(x) s.t. $g(x) \leq 0$
- x_0 is a local optimum iff

- In a convex program,

An optimality criterion

- Suppose f(x) is differentiable, convex
- Then x* = arg min f(x) iff
- What about x* = arg min f(x) s.t. $x \in C$?

Proof of criterion

Example: logistic regression

- Data $x_i \in R$, $y_i \in \{0,1\}$
- $P(y_i = 1 \mid x_i, w, b) = s(w^T x_i + b)$
  $s(z) = 1/(1+\exp(-z))$

- $\max_{wb} P(w, b)\, P(y \mid x, w, b) =$
  $\max_{wb} P(w, b) \prod_i P(y_i \mid x_i, w, b) =$

Ex: minimize top eigenvalue

- Suppose $A_i \in S^{n*n}$ for $i = \{1, 2, \ldots\}$
- $\min_{\alpha,A}\ \lambda_1(A)$ s.t.
  $A = \alpha_1 A_1 + \alpha_2 A_2 + \ldots$

Ex: minimum volume ellipsoid

- Given points $x_1, x_2, \ldots, x_k$
- $\min_{A, x_C}$ vol(A) s.t.
  $(x_i - x_C)^T A (x_i - x_C) \le 1 \quad i = 1, \ldots, k$
  $A \in S^{n*n}$
  $A \succcurlyeq 0$

Schur complement

- Symmetric block matrix M =
- Schur complement is S =
- $M \succcurlyeq 0$ iff

Back to min-volume ellipsoid

- $\max_{A, x_C} \; \log |A|$ s.t.
  $(x_i - x_C)^T A (x_i - x_C) \leq 1 \quad i = 1, \ldots, k$
  $A = A^T, A \succcurlyeq 0$
- $\max_{A, B, x_C, z} \; \log |A|$ s.t.