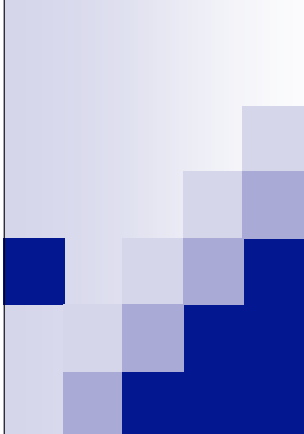




Delayed Column Generation (aka variable generation)

Optimization - 10725
Carlos Guestrin
Carnegie Mellon University

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Why constraint generation converges



- LP with many many constraints:
 - Solve with subset of constraints:
 - (also called “cutting planes”)
 - Relaxed problem, bound on objective:
- If solution x_Ω is feasible wrt all constraints:
- If solution x_Ω is infeasible wrt all constraints:

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Constraint generation and duality

- Primal problem with many constraints:

$$\begin{aligned} \max_x \quad & \sum_j b_j x_j \\ \text{s.t.} \quad & \sum_j a_{ij} x_j \leq c_i, \quad \forall i \in \mathcal{I} \end{aligned}$$

- Constraint generation: find most important constraints
- What's the dual equivalent?
- Dual:

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Column generation (aka variable generation)

- Dual problem:
$$\begin{aligned} \min_y \quad & \sum_{i \in \mathcal{I}} c_i y_i \\ \text{s.t.} \quad & \sum_{i \in \mathcal{I}} a_{ij} y_i = b_j, \quad \forall j \in 1, \dots, m \\ & y_i \geq 0, \quad \forall i \in \mathcal{I} \end{aligned}$$
- Many many variables!!
- At optimal basic feasible solution
 - Most variables are zero
- Idea:
 - Set most variables to zero
 - Solve problem with other variables:
 - Incrementally increase sets of non-zero variables

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Solving problem with subset of variables

- Solve problem with subset of variables

$$\begin{array}{ll} \min_y & \sum_{i \in \Omega} c_i y_i \\ \text{s.t.} & \sum_{i \in \Omega} a_{ij} y_i = b_j, \quad \forall j \in 1, \dots, m \\ & y_i \geq 0, \quad \forall i \in \Omega \end{array}$$
- Rest of variables set to zero
- Questions:
 - ☐ How do we decide what variables to use?
 - ☐ How do we decide when we are done?

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What variables should we add?

- Same as simplex

$$\begin{array}{ll} \min_y & \sum_{i \in \Omega} c_i y_i \\ \text{s.t.} & \sum_{i \in \Omega} a_{ij} y_i = b_j, \quad \forall j \in 1, \dots, m \\ & y_i \geq 0, \quad \forall i \in \Omega \end{array}$$
- ☐
- Solve problem with variables Ω
 - ☐ At optimal basic feasible solution, set of basic variables B
- Find submatrix corresponding to basic variables A_B
 - ☐ Cost of these variables c_B
- Reduced cost for each potential new variable x_i , for $i \in I$:
 - ☐ If all are positive?
 - ☐ Otherwise:
- Guaranteed to converge to optimal solution

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Column generation summary

- Dual of constraint generation
- Also useful for problems with infinitely many variables
- Some problems
 - Have efficient separation oracles
 - In these, constraint generation is useful
 - Have efficient variable generation oracles
 - In these, column generation is useful
- Both methods can be useful in polynomially large problems
 - E.g., when constraint matrix is too large to fit in memory
 - By incrementally solving the problem, bound amount of memory needed at each iteration
- If you have many many variables and constraints
 - Can use a combination of constraint and column generation