Readings:
Review: K&F: *2.1*, 2.2, 2.3
K&F: 3.1

Introduction

Graphical Models – 10708
Carlos Guestrin
Carnegie Mellon University

September 8th, 2008

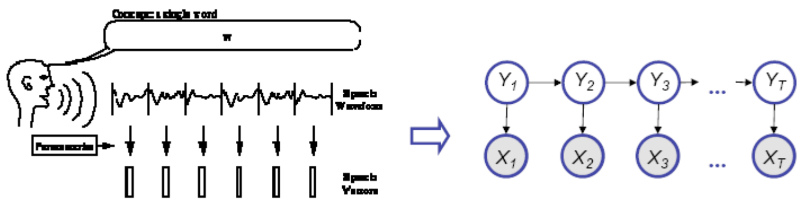
This Course

One of the most exciting developments in machine learning (knowledge representation, AI, EE, Stats,…) in the last two (or three, or more) decades…

My expectations are already high… ☺

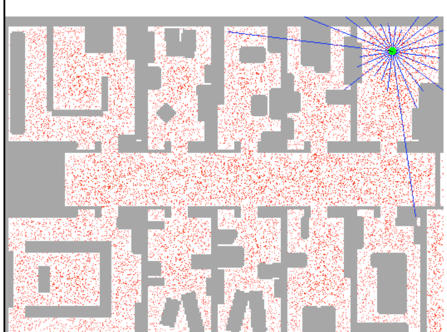
Speech recognition

Hidden Markov models and their generalizations

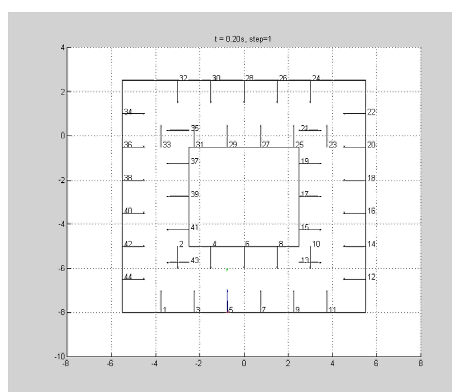


Tracking and robot localization

Kalman Filters



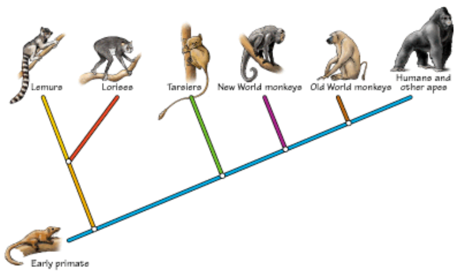
[Fox et al.]

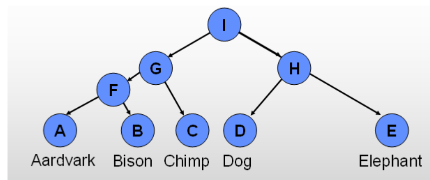


[Funiak et al.]

Evolutionary biology

Bayesian networks

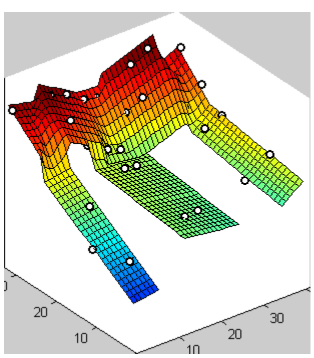


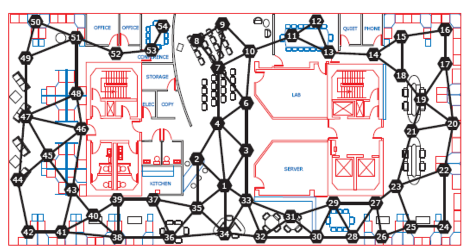


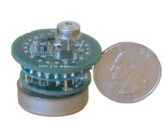
[Friedman et al.]

Modeling sensor data

Undirected graphical models



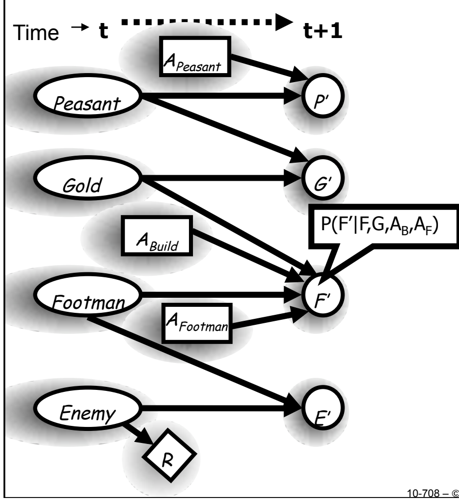


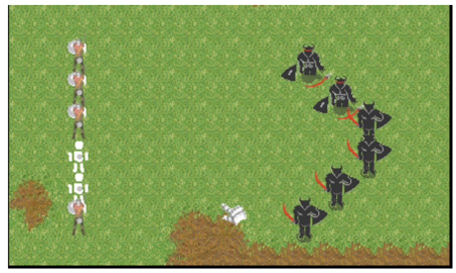


[Guestrin et al.]

Planning under uncertainty

Dynamic Bayesian networks
Factored Markov decision problems

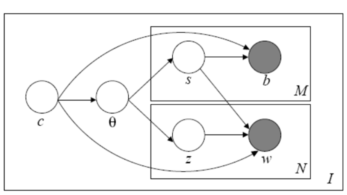


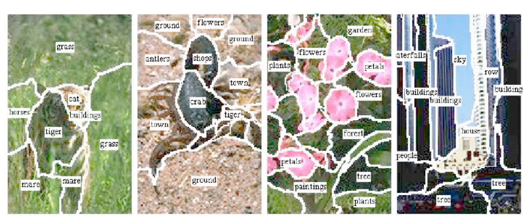


[Guestrin et al.]

Images and text data

Hierarchical Bayesian models

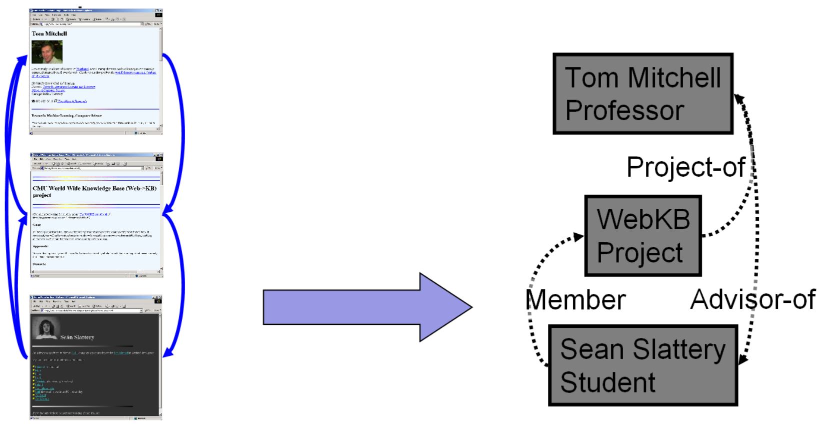




[Barnard et al.]

Structured data (text, webpages,…)

Probabilistic relational models



[Koller et al.]

And many

many

many

many

many

more…

Syllabus

- Covers a wide range of Probabilistic Graphical Models topics – from basic to state-of-the-art
- You will learn about the methods you heard about:
  - Bayesian networks, Markov networks, factor graphs, conditional random fields, decomposable models, junction trees, parameter learning, structure learning, semantics, exact inference, variable elimination, context-specific independence, approximate inference, sampling, importance sampling, MCMC, Gibbs, variational inference, loopy belief propagation, generalized belief propagation, Kikuchi, Bayesian learning, missing data, EM, Chow-Liu, structure search, IPF for tabular MRFs, Gaussian and hybrid models, discrete and continuous variables, temporal and template models, hidden Markov Models, Forwards-Backwards, Viterbi, Baum-Welch, Kalman filter, linearization, switching Kalman filter, assumed density filtering, DBNs, BK, Relational probabilistic models, Causality,…
- Covers algorithms, theory and applications
- **It's going to be fun and hard work ☺**

Prerequisites

- 10-701 – Machine Learning, especially:
  - Probabilities
    - Distributions, densities, marginalization…
  - Basic statistics
    - Moments, typical distributions, regression…
- Algorithms
  - Dynamic programming, basic data structures, complexity…
- Programming
  - Matlab will be very useful
- We provide some background, but the class will be fast paced
- Ability to deal with "abstract mathematical concepts"

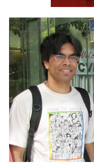
Review Sessions

- Very useful!
  - Review material
  - Present background
  - Answer questions
- Thursdays, 5:00-6:20 in Wean Hall 5409
- First recitation is **this Thursday**
  - Review of probabilities & statistics
- Sometimes this semester: Especial recitations most likely on Mondays 5:30-7pm
  - Cover special topics that we can't cover in class
  - These are optional, but you are here to learn… ☺
- Do ~~we need a Matlab review session~~?

Staff

- Two Great TAs: Great resource for learning, interact with them!
  - Amr Ahmed <amahmed@cs.cmu.edu>,

  - Dhruv Batra <batradhruv@cmu.edu>



- Administrative Assistant
  - Michelle Martin <michelle324@cs.cmu.edu>, Wean 4619, x8-5527

http:// www.cs.cmu.edu/~guestrin/Class/10708

First Point of Contact for HWs

- To facilitate interaction, a TA will be assigned to each homework question – This will be your "first point of contact" for this question
  - But, you can always ask any of us
- For e-mailing instructors, always use:
  - 10708-instr@cs.cmu.edu
- For announcements, subscribe to:
  - 10708-announce@cs
  - https://mailman.srv.cs.cmu.edu/mailman/listinfo/10708-announce
- We will also use a discussion group:
  - Post your questions, discuss projects, etc
  - Be nice… ☺
  - Don't give away any answers… ☺
  - http://groups.google.com/group/10708-f08

Text Books

- *Primary*: Daphne Koller and Nir Friedman, **Structured Probabilistic Models**, in preparation. These chapters are part of the course reader. You can purchase one from Michelle Martin
- *Secondary*: M. I. Jordan, **An Introduction to Probabilistic Graphical Models,** in preparation. Copies of selected chapters will be made available.

Grading

- 5 homeworks (50%)
  - First one goes out next Wednesday!
  - Homeworks are long and hard ☺
    - please, please, please, please, please, please start early!!!
- Final project (30%)
  - Done individually or in pairs
  - Details out soon
  - Proposals due October 6^{th}
- Final (20%)
  - Take home, out Dec. 3^{rd}
  - Due Dec. 10^{th} at NOON (hard deadline)

Homeworks

- Homeworks are hard, start early ☺
- Due in the beginning of class
- 3 late days for the semester
- After late days are used up:
  - Half credit within 48 hours
  - Zero credit after 48 hours
- All homeworks **must be handed in**, even for zero credit
- Late homeworks handed in to Michelle Martin, WEH 4619

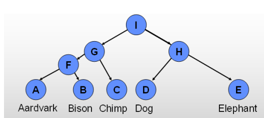
- Collaboration
  - You may **discuss** the questions
  - Each student writes their own answers
  - Write on your homework anyone with whom you collaborate

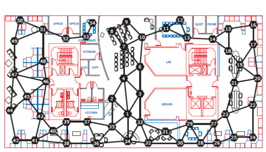
- **IMPORTANT**:
  - **We may use some material from previous years or from papers for the homeworks. Unless otherwise specified, please only look at the readings when doing your homework → You are taking this advanced graduate class because you want to learn, so this rule is self-enforced ☺**

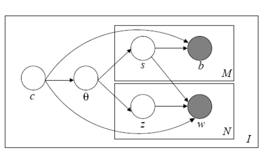
Enjoy!

- NO CLASS THIS WEDNESDAY 9/10
- Probabilistic graphical models are having significant impact in science, engineering and beyond
- This class should give you the basic foundation for applying GMs and developing new methods
- The fun begins…

What are the fundamental questions of graphical models?

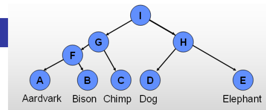


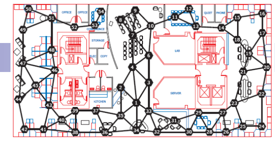




- Representation:
  - What are the types of models?
  - What does the model mean/imply/ assume? (Semantics)
- Inference:
  - How do I answer questions/queries with my model?
- Learning:
  - What model is the right for my data?

More details???





- Representation:
  - Graphical models represent exponentially large probability distributions compactly
  - **Key concept**: *Conditional Independence*
- Inference:
  - What is the probability of X given some observations?
  - What is the most likely explanation for what is happening?
  - What decisions should I make?
- Learning:
  - What are the right/good parameters for the model?
  - How do I obtain the structure of the model?

Where do we start?

- From Bayesian networks
- "Complete" BN presentation first
  - Representation
  - Exact inference
  - Learning
  - Only discrete variables for now
- Later in the semester
  - Undirected models
  - Approximate inference
  - Continuous
  - Temporal models
  - And more…
- Class focuses on fundamentals – Understand the foundation and basic concepts

Today

- Probabilities
- Independence
- Two nodes make a BN
- Naïve Bayes

- Should be a review for everyone – Setting up notation for the class

Random variable

- Probability distributions usually defined by events
- Events are complicated – we think about attributes
  10708
  - Age, Grade, HairColor
- Random variables formalize attributes:
  - Grade=A —— shorthand for event $\{\omega \in \Omega: f_{Grade}(\omega) = A\}$

- Properties of random vars, X:
  - Val(X) = possible values of random var X
  - For discrete (categorical): $\sum_{i=1\ldots|Val(X)|} P(X=x_i) = 1$
  - For continuous: $\int_x p(X=x)dx = 1$
  - $P(x) \geq 0$

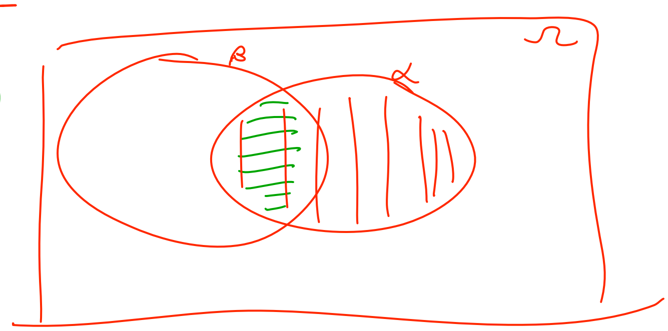
Interpretations of probability – A can of worms!

- Frequentists
  - P(α) is the frequency of α in the limit
  - Many arguments against this interpretation
    - What is the frequency of the event “it will rain tomorrow”?
- Subjective interpretation
  - P(α) is my degree of belief that α will happen
  - What the …. does “degree of belief mean?
  - If I say P(α)=0.8, then I am willing to bet!!!
- For this class, we (mostly) don’t care what camp you are in

Conditional probabilities

- After learning that α is true, how do we feel about β?
- P(β|α)

$$= \frac{P(\beta \wedge \alpha)}{P(\alpha)}$$



Two of the most important rules of the semester: 1. The chain rule

- $P(\alpha \cap \beta)=P(\alpha)P(\beta|\alpha)$

- More generally:
  - $P(\alpha_1 \cap \ldots \cap \alpha_k)= P(\alpha_1)\, P(\alpha_2|\alpha_1) \cdots P(\alpha_k|\alpha_1 \cap \ldots \cap \alpha_{k-1})$

Two of the most important rules of the semester: 2. Bayes rule

likelihood

prior

- $$P(\alpha \mid \beta) = \frac{P(\beta \mid \alpha)P(\alpha)}{P(\beta)}$$

posterior

normalization constant

- More generally, external event γ:
  - $$P(\alpha \mid \beta \cap \gamma) = \frac{P(\beta \mid \alpha \cap \gamma)P(\alpha \mid \gamma)}{P(\beta \mid \gamma)}$$

Most important concept: a) Independence

- α and β ***independent***, if $P(\beta|\alpha)=P(\beta)$
  - $P \rightarrow (\alpha \perp \beta)$

- **Proposition:** α and β *independent* if and only if $P(\alpha \cap \beta)=P(\alpha)P(\beta)$

Most important concept: b) Conditional independence

- Independence is rarely true, but conditionally…

- α and β ***conditionally independent*** given γ if $P(\beta|\alpha \cap \gamma)=P(\beta|\gamma)$
  - $P \models (\alpha \perp \beta \mid \gamma)$

Proposition: $P \models (\alpha \perp \beta \mid \gamma)$ if and only if $P(\alpha \cap \beta \mid \gamma)=P(\alpha \mid \gamma)P(\beta \mid \gamma)$

Joint distribution, Marginalization

- Two random variables – Grade & Intelligence

$$P(G, I) =$$

G \ I	VH	H
A	.7	.1
B	.15	0.05

- Marginalization – Compute marginal over single var

$$P(G=B) = \underset{.15}{P(G=B, I=VH)} + \underset{.05}{P(G=B, I=H)}$$

$$= 0.2$$

Marginalization – The general case

Notation

assignment

→ $x_j \in Val(X_j)$

→ X_j ← random var

short hand for $X_j = x_j$ for all values

- Compute marginal distribution $P(X_i)$:

$X_1 \dots X_n$

$$P(X_1, X_2, \dots, X_i) = \sum_{x_{i+1}, \dots, x_n} P(X_1, X_2, \dots, X_i, x_{i+1}, \dots, x_n)$$

$$P(X_i) = \sum_{x_1, \dots, x_{i-1}} P(x_1, \dots, x_{i-1}, X_i)$$

if binary vars: 2^{n-1} terms

Basic concepts for random variables

- Atomic outcome: assignment $x_1,\ldots,x_n$ to $X_1,\ldots,X_n$
- Conditional probability: $P(X,Y)=P(X)P(Y|X)$
- Bayes rule: $P(X|Y)= \dfrac{P(Y|X)\,P(X)}{P(Y)}$
- Chain rule:
  - $P(X_1,\ldots,X_n) = P(X_1)P(X_2|X_1)\ldots P(X_n|X_1,\ldots,X_{n-1})$

Conditionally independent random variables

bold face

- **Sets** of variables **X**, **Y**, **Z**
- $\cancel{X}$ **X** is independent of Y given Z if
  - $P \to (\mathbf{X}=\mathbf{x} \perp \mathbf{Y}=\mathbf{y} | \mathbf{Z}=\mathbf{z})$, $\forall\ \mathbf{x} \in Val(\mathbf{X})$, $\mathbf{y} \in Val(\mathbf{Y})$, $\mathbf{z} \in Val(\mathbf{Z})$

    $P(X=x \mid Y=y, Z=z) = P(X=x \mid Z=z)$
- Shorthand:
  - **Conditional independence:** $P \to (\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})$
  - For $P \to (\mathbf{X} \perp \mathbf{Y} \mid \emptyset)$, write $P \to (\mathbf{X} \perp \mathbf{Y})$
- **Proposition:** P satisfies $(\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})$ if and only if
  - $P(\mathbf{X},\mathbf{Y}|\mathbf{Z}) = P(\mathbf{X}|\mathbf{Z})\ P(\mathbf{Y}|\mathbf{Z})$

Properties of independence

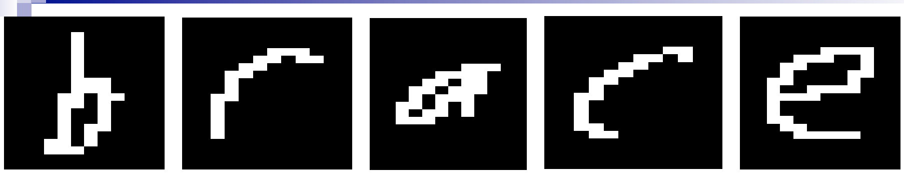
- **Symmetry:**
  - (**X** ⊥ **Y** | **Z**) ⇒ (**Y** ⊥ **X** | **Z**)
- **Decomposition:**
  - (**X** ⊥ **Y**,**W** | **Z**) ⇒ (**X** ⊥ **Y** | **Z**)
- **Weak union:**
  - (**X** ⊥ **Y**,**W** | **Z**) ⇒ (**X** ⊥ **Y** | **Z**,**W**)
- **Contraction:**
  - (**X** ⊥ **W** | **Y**,**Z**) & (**X** ⊥ **Y** | **Z**) ⇒ (**X** ⊥ **Y**,**W** | **Z**)
- **Intersection:**
  - (**X** ⊥ **Y** | **W**,**Z**) & (**X** ⊥ **W** | **Y**,**Z**) ⇒ (**X** ⊥ **Y**,**W** | **Z**)
  - Only for positive distributions!
  - $P(\alpha)>0, \forall\alpha, \alpha\neq\emptyset$
- **Notation:** *I(P)* – independence properties entailed by *P*

Bayesian networks

- One of the most exciting recent advancements in statistical AI
- Compact representation for exponentially-large probability distributions
- Fast marginalization too
- Exploit conditional independencies

Handwriting recognition

Handwriting recognition 2



$$\begin{tikzcd} {} & {} & {} & {} & {} \\ X_1 \arrow[u] \arrow[r] & X_2 \arrow[u] \arrow[r] & X_3 \arrow[u] \arrow[r] & X_4 \arrow[u] \arrow[r] & X_5 \arrow[u] \end{tikzcd}$$

Webpage classification

→ Company home page

vs

Personal home page

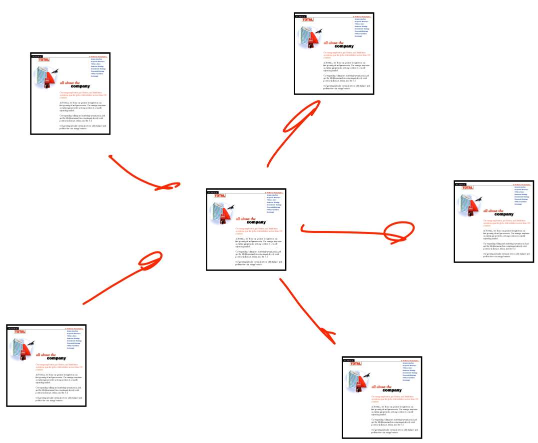
vs

Univeristy home page

vs

…

Webpage classification 2



Let's start on BNs…

- Consider $P(X_i)$
  - Assign probability to each $x_i \in Val(X_i)$
  - Independent parameters $|Val(x_i)| = k$

    $k-1$
- Consider $P(X_1,\ldots,X_n)$
  - How many independent parameters if $|Val(X_i)|=k$?

    $k^n - 1$

Same thing w. fewer params } BN

What if variables are independent?

- What if variables are independent?
  - $(X_i \perp X_j), \forall$ i,j
  - Not enough!!! (See homework 1 ☺)
  - Must assume that $(\mathbf{X} \perp \mathbf{Y}), \forall$ **X**,**Y** subsets of $\{X_1,\ldots,X_n\}$

    $X_1 X_3 \perp X_7 X_{14}$

    $X_1 X_{14} \perp X_3 X_2 X_5$
- Can write
  - $P(X_1,\ldots,X_n) = \prod_{i=1\ldots n} P(X_i)$

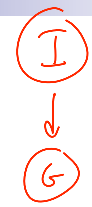
    BN w. no edges

    X_1 X_2 X_3 ... X_n
- How many independent parameters now?

  $n.(k-1)$

Conditional parameterization – two nodes

- Grade is determined by Intelligence



$P(I) =$

VH	H
.85	.15

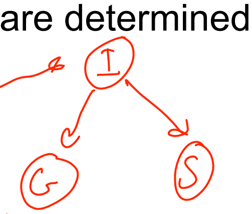
$P(G|I) =$

G \ I	VH	H
A	.9	.5
B	.1	.5

$P(I=VH, G=B)$
$= P(I=VH)\,P(G=B|I=VH)$
$= .85 \times .1$
$= 0.085$

Conditional parameterization – three nodes

- Grade and SAT score are determined by Intelligence
- $(G \perp S \mid I)$



$P(I)$

$P(G|I)$ $P(S|I)$

The naïve Bayes model – Your first real Bayes Net

- Class variable: C
- Evidence variables: $X_1,\ldots,X_n$
- assume that $(\mathbf{X} \perp \mathbf{Y} \mid C)$, $\forall$ $\mathbf{X},\mathbf{Y}$ subsets of $\{X_1,\ldots,X_n\}$

What you need to know

- Basic definitions of probabilities
- Independence
- Conditional independence
- The chain rule
- Bayes rule
- Naïve Bayes

Next class

- We've heard of Bayes nets, we've played with Bayes nets, we've even used them in your research
- Next class, we'll learn the semantics of BNs, relate them to independence assumptions encoded by the graph