Bayes Network

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Recap

- A path $x_1 - x_2 - \cdots - x_k$ is an active trail when variables $z \subseteq \{x_1, \ldots, x_n\}$ are observed if for EACH consecutive triplet in the trail:
  - $x_{i-1} \rightarrow x_i \rightarrow x_{i+1}$ and $x_i$ is NOT observed
  - $x_{i-1} \leftarrow x_i \leftarrow x_{i+1}$ and $x_i$ is NOT observed
  - $x_{i-1} \leftarrow x_i \rightarrow x_{i+1}$ and $x_i$ is NOT observed
  - $x_{i-1} \rightarrow x_i \leftarrow x_{i+1}$ and $x_i$ is observed, or one of its descendents is observed
Recap

☐ Theorem: variables \( x_i \) and \( x_j \) are independent given \( z \subseteq \{x_1, \ldots, x_n\} \) if there is \textit{NO active trail} between \( x_i \) and \( x_j \) when variables \( z \subseteq \{x_1, \ldots, x_n\} \) are observed.

☐ In other words, every trail between \( x_i \) and \( x_j \) is \textit{NON-active} when \( z \subseteq \{x_1, \ldots, x_n\} \) are observed.
NON-Active Trail

- There exists a node $V$, such that
  - $V$ is observed, and
  - ![Diagram of node $V$ with an arrow pointing towards another node](image)

- OR there exists a node $V$, such that
  - $V$ is observed, and
  - ![Diagram of node $V$ with an arrow pointing away from another node](image)

- OR there exists a node $V$, such that
  - **Neither $V$ nor** any of its descendant is observed, and
  - ![Diagram of V-structure](image) (V-structure)
Examples

- Trail from A to B

- Z=\[\]  Active
- Z=[C]  Non-Active
- Z=[D]  Active
- Z=[C, D]  Non-Active

- Z=\[\]  Non-Active
- Z=[C]  Non-Active
- Z=[D]  Non-Active
- Z=[C, E]  Active
- Z=[C, D, E]  Non-Active
Test Your Understanding

- I<C, {}, D>?
- I<C, {A}, D>?
- I<C, {A, B}, D>?
- I<C, {A, B, J}, D>?
- I<C, {A, B, E, J}, D>?  

✓ ✔ ✗ ✗
Test Your Understanding

- $I\langle B, \{E, F\} \rangle$? ×
- $I\langle G, \{F, I, K\} \rangle$? ✓
- $I\langle D, \{G, I\} \rangle$? ✓
- $I\langle B, \{A, F\}, H \rangle$? ✓
Inference

\[ P(B = T | C = T) \]
Inference

- \[ P(B = T | C = T) = \frac{P(B = T, C = T)}{P(C = T)} \]
- \[ P(B = T, C = T) = \sum_A P(A, B = T, C = T) \]
  \[ = \sum_A P(A) P(B = T | A) P(C = T | A) \]
  \[ = 0.2 \times 0.3 \times 0.37 + 0.8 \times 0.25 \times 0.21 \]
  \[ = 0.0642 \]
Inference

- \( P(C = T) = P(B = T, C = T) + P(B = F, C = T) \)

- \( P(B = F, C = T) = \sum_A P(A, B = F, C = T) \)
  
  \[ = \sum_A P(A) P(B = F \mid A) P(C = T \mid A) \]
  
  \[ = 0.2 \times 0.7 \times 0.37 + 0.8 \times 0.75 \times 0.21 \]
  
  \[ = 0.1778 \]
Inference

\[ P(B = T | C = T) = \frac{P(B = T, C = T)}{P(C = T)} \]

\[ = \frac{P(B = T, C = T)}{P(B = T, C = T) + P(B = F, C = T)} \]

\[ = \frac{0.0642}{0.0642 + 0.1778} \]

\[ = 0.2653 \]