**More details:**
General: http://www.learning-with-kernels.org/
Example of more complex bounds:
http://www.research.ibm.com/people/t/tzhang/papers/jmlr02_cover.ps.gz

# PAC-learning, VC Dimension and Margin-based Bounds


Machine Learning – 10701/15781
Carlos Guestrin
Carnegie Mellon University


March 6th, 2006

# Announcements 1

- Midterm on Wednesday
  - open book, texts, notes,…
  - no laptops
  - bring a calculator

Review session today at 5pm

NSH 3305

# Announcements 2

- Final project details are out!!!
  - http://www.cs.cmu.edu/~guestrin/Class/10701/projects.html
  - Great opportunity to apply ideas from class and learn more
  - Example project:
    - Take a dataset
    - Define learning task
    - Apply learning algorithms
    - Design your own extension
    - Evaluate your ideas
  - many of suggestions on the webpage, but you can also do your own

talk to us first.

- Boring stuff:
  - Individually or groups of two students
  - It's worth 20% of your final grade
  - You need to submit a one page proposal on Wed. 3/22 (just after the break)
  - A 5-page initial write-up (milestone) is due on 4/12 (20% of project grade)
  - An 8-page final write-up due 5/8 (60% of the grade)
  - A poster session for all students will be held on Friday 5/5 2-5pm in NSH atrium (20% of the grade)
  - You can use late days on write-ups, each student in team will be charged a late day per day.

page limits are strict...

- MOST IMPORTANT:

Have some Fun!! :)

# What now…

- We have explored **many** ways of learning from data
- But…
  - How good is our classifier, really?
  - How much data do I need to make it "good enough"?

Learning Theory

# How likely is learner to pick a bad hypothesis

- Prob. *h* with $\text{error}_{\text{true}}(h) \geq \varepsilon$ gets *m* data points right

- There are *k* hypothesis consistent with data
  - How likely is learner to pick a bad one?

P(at least one of the k was bad and it got lucky) ?

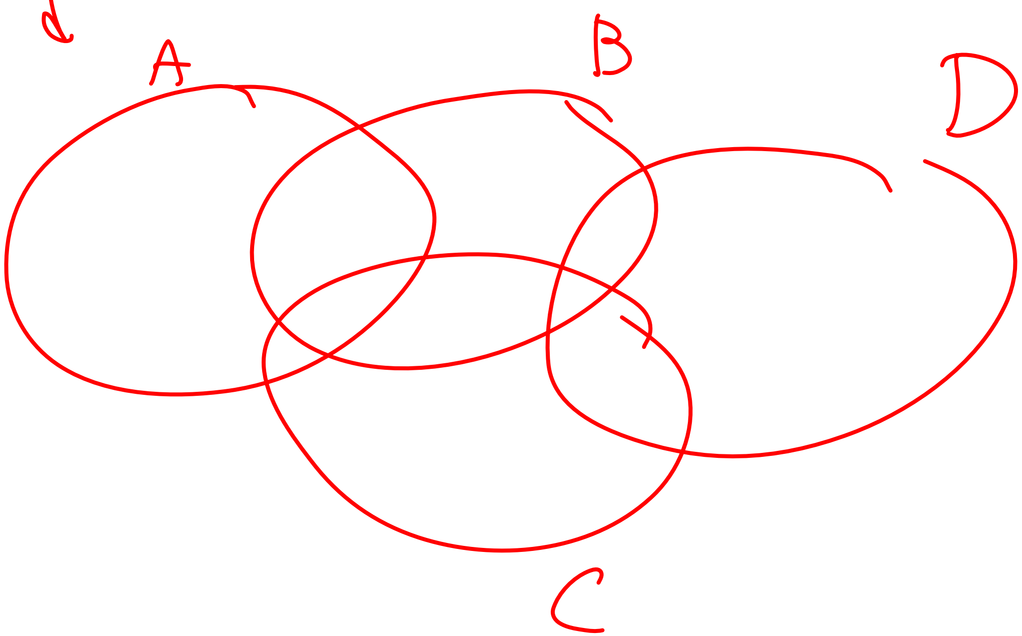
P(one got lucky) $\leq \underbrace{(1-\varepsilon)^m}$ k

# Union bound

bad hyp. got all m right

- P(A or B or C or D or …) $\leq P(A) + P(B) + P(C) + \cdots$

area



# How likely is learner to pick a bad hypothesis

- Prob. $h$ with $\text{error}_{\text{true}}(h) \geq \varepsilon$ gets $m$ data points right

- There are $k$ hypothesis consistent with data
  - How likely is learner to pick a bad one?

P( h1 bad & got lucky or h2 bad & got lucky . or h3 ... )

$\leq P(h_1 \text{ bad \& lucky}) + P(h_2 \text{ bad \& lucky}) + P(h_3 \ldots) + \ldots$

$\leq (1-\varepsilon)^m$

how big is K

$K \leq |H|$ (loose bound!)

$1-\varepsilon \leq e^{-\varepsilon}$ (make eq. simpler)

$\leq K(1-\varepsilon)^m$

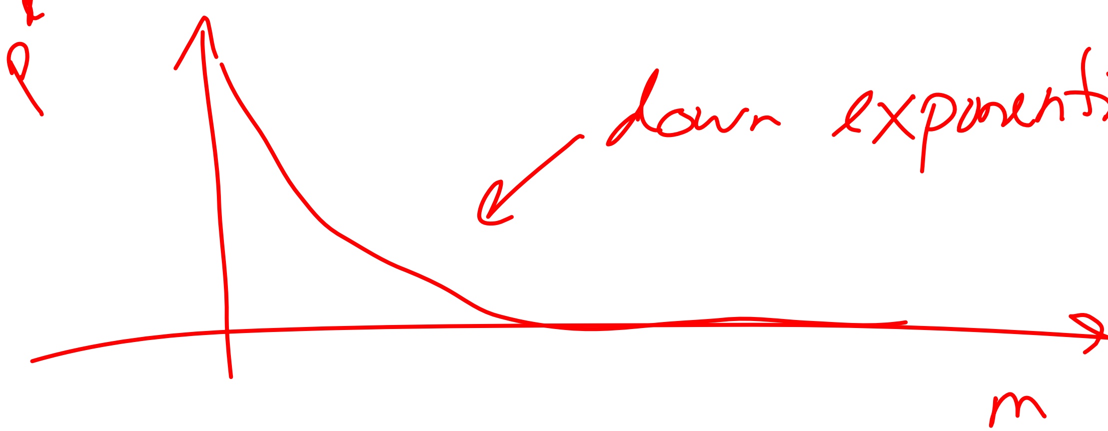
$\leq |H| e^{-\varepsilon m}$

# Review: Generalization error in finite hypothesis spaces [Haussler '88]

- ***Theorem***: Hypothesis space $H$ finite, dataset $D$ with $m$ i.i.d. samples, $0 < \varepsilon < 1$ : for any learned hypothesis $h$ that is consistent on the training data:

$$P(\text{error}_{\cancel{\mathcal{X}}\,\text{true}}(h) > \epsilon) \leq |H|e^{-m\epsilon} \leq \delta$$

0.01




down exponentially fast!

# Using a PAC bound

- Typically, 2 use cases:
  - 1: Pick ε and δ, give you *m*
  - 2: Pick m and δ, give you ε

$$P(\text{error}_{\cancel{x}}(h) > \epsilon) \leq |H|e^{-m\epsilon}$$

true

(1) $P \leq |H| e^{-m\varepsilon} \leq \delta$

$\ln|H| - m\varepsilon \leq \ln\delta$ less

$\Rightarrow m \geq \frac{1}{\varepsilon}\left(\ln|H| + \ln\frac{1}{\delta}\right)$

smaller $\delta$ ⇒ need more data

smaller ε more data

not as much data as you may think

case 2 $\delta \leq |H| e^{-m\varepsilon}$

$\ln\delta \leq \ln|H| - m\varepsilon$

$\varepsilon \leq \frac{1}{m}\left(\ln|H| + \ln\frac{1}{\delta}\right)$

learn h

true error ≤

before you run the algorithm

# Review: Generalization error in finite hypothesis spaces [Haussler '88]

- ***Theorem***: Hypothesis space *H* finite, dataset *D* with *m* i.i.d. samples, $0 < \varepsilon < 1$ : for any learned hypothesis *h* that is consistent on the training data:

$$P(\text{error}_{\mathcal{X}}(h) > \epsilon) \leq |H| e^{-m\epsilon}$$

if I can always learn a consistent classifier

then

**Even if *h* makes zero errors in training data, may make errors in test**

# Limitations of Haussler '88 bound

$$P(\text{error}_{\cancel{x}\,\text{true}}(h) > \epsilon) \leq |H|e^{-m\epsilon}$$

1. - Consistent classifier

there may not be such h in class!

2. - Size of hypothesis space

bound depends on |H|

→ really really large?

→ infinite? w continuous

# Simpler question: What's the expected error of a hypothesis?

- The error of a hypothesis is like estimating the parameter of a coin!

flip coin $m$ times

(train error)

true error ← $\theta$

I don't know it

$\theta$ v. $\hat{\theta}$

$\hat{\theta}$

- Chernoff bound: for *m* i.i.d. coin flips, $x_1,\ldots,x_m$, where $x_i \in \{0,1\}$. For $0<\varepsilon<1$:

$$\boxed{P\left(\theta - \frac{1}{m}\sum_i x_i > \epsilon\right) \leq e^{-2m\epsilon^2}}$$

truth

train error

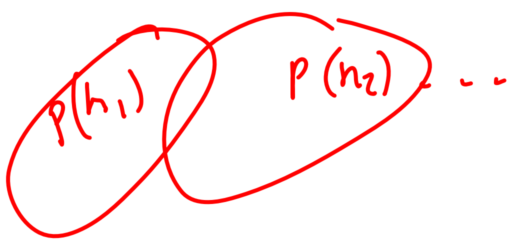
sample average

# But we are comparing many hypothesis: **Union bound**

For each hypothesis $h_i$:

$$P\left(\text{error}_{true}(h_i) - \text{error}_{train}(h_i) > \epsilon\right) \leq e^{-2m\epsilon^2}$$

What if I am comparing two hypothesis, $h_1$ and $h_2$?




learner is going to compare all of $h_i$'s

$$P(\exists_i\ \text{error}_{true}(h_i) - \text{error}_{train}(h_i) > \varepsilon) \leq |H|\, e^{-2m\varepsilon^2} \quad \delta$$

# Generalization bound for |H| hypothesis

- ***Theorem***: Hypothesis space $H$ finite, dataset $D$ with $m$ i.i.d. samples, $0 < \varepsilon < 1$ : for any learned hypothesis $h$:

$$P\left(\text{error}_{true}(h) - \text{error}_{train}(h) > \epsilon\right) \leq |H| e^{-2m\epsilon^2}$$

$2m\varepsilon^2 = 20$ not as good !!

side note: Haussler's bound for consistent $h$:

$P \leq |H| e^{-m\varepsilon}$

$\varepsilon = 0.1$

$m = 1000$

$\Rightarrow m \cdot \varepsilon = 100$

# PAC bound and Bias-Variance tradeoff

$$P\left(\mathrm{error}_{true}(h) - \mathrm{error}_{train}(h) > \epsilon\right) \leq |H| e^{-2m\epsilon^2}$$

**or, after moving some terms around, with probability at least 1-δ:**

$$\mathrm{error}_{true}(h) \leq \mathrm{error}_{train}(h) + \sqrt{\frac{\ln |H| + \ln \frac{1}{\delta}}{2m}}$$

want to minimize

| "bias" | "variance" | |
|---|---|---|
| ↑ larger | ↓ smaller | \|H\| small |
| ↓ smaller | ↑ larger | \|H\| large |

- **Important: PAC bound holds for all *h*, but doesn't guarantee that algorithm finds best *h*!!!**

# What about the size of the hypothesis space?

$$m \geq \frac{1}{2\epsilon^2}\left(\ln|H| + \ln\frac{1}{\delta}\right)$$

need this amount

- How large is the hypothesis space? $|H|$

$\ln|H|$ ?

# Boolean formulas with *n* binary features

$$m \geq \frac{1}{2\epsilon^2}\left(\ln|H| + \ln\frac{1}{\delta}\right)$$

what's ln|H|?

tabular representation

| $X_1$ | $X_2$ | $X_3$ | $X_4$ | Y |
|---|---|---|---|---|
| T | T | T | T | 0/1 |
| T | T | T | F | 0/1 |
| T | T | F | T | 0/1 |
| T | T | F | F | ⋮ |

$2^n$ rows each 2 possibil.

$|H| = 2^{2^n}$ (reallyreally large)

$\ln|H| = 2^n \ln 2$ too large

conjunctions:

$h_1 = X_1 \wedge X_2 \wedge X_7$

$h_2 = X_2 \wedge \neg X_5 \wedge X_8 \ldots$

n attrib..

$\langle \{\emptyset, 1, \neg\}, \{\emptyset, 1, \neg\}, \ldots \rangle$

$|H| = 3^n$ (really large)

$\ln|H| = n \ln 3$ "small"

binary features

# Number of decision trees of depth k

$$m \ge \frac{1}{2\epsilon^2}\left(\ln|H| + \ln\frac{1}{\delta}\right)$$

what's $\ln|H|$

Recursive solution

Given *n* attributes

$H_k$ = Number of decision trees of depth k

$H_0$ =2

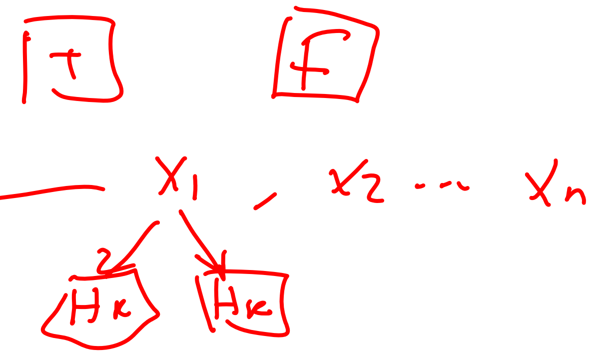
$H_{k+1}$ = (#choices of root attribute) *

(# possible left subtrees) *

(# possible right subtrees)

= n * $H_k$ * $H_k$

(upper bound... may not use same feature twice)




Write $L_k = \log_2 H_k$

$L_0 = 1$

$L_{k+1} = \log_2 n + 2L_k$

So $L_k = (2^k-1)(1+\log_2 n) +1$

# PAC bound for decision trees of depth k

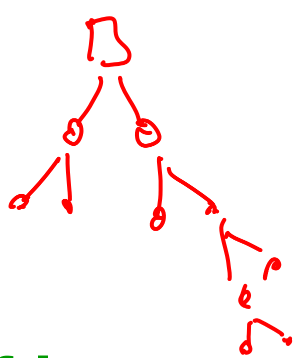
plug into bound

$$m \geq \frac{\ln 2}{2\epsilon^2}\left((2^k - 1)(1 + \log_2 n) + 1 + \ln\frac{1}{\delta}\right)$$

- Bad!!!
  - Number of points is exponential in depth!

- But, for *m* data points, decision tree can't get too big…

DT:



after m leaves
not worth
splitting any more!

**Number of leaves never more than number data points**

# Number of decision trees with k leaves

$$m \geq \frac{1}{2\epsilon^2}\left(\ln|H| + \ln\frac{1}{\delta}\right)$$

$H_k$ = Number of decision trees with k leaves

$H_0 = 2$

bound

n features for root

k ways to split k between left & right

$x_i$

$$H_{k+1} = n \sum_{i=1}^{k} H_i H_{k+1-i}$$

i leaves left

rest on right.

10 leaves m-10 leaves

11 m-11

12 m-12 ...

quick bound

**Loose bound:**

$$H_k \leq n^{k-1}(k+1)^{2k-1}$$

**Reminder:**

$$|\text{DTs depth } k| = 2 * (2n)^{2^k - 1}$$

# PAC bound for decision trees with k leaves – Bias-Variance revisited

$$H_k = n^{k-1}(k+1)^{2k-1}$$

$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{\ln|H| + \ln\frac{1}{\delta}}{2m}}$$

$$\boxed{\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{(k-1)\ln n + (2k-1)\ln(k+1) + \ln\frac{1}{\delta}}{2m}}}$$

| | | |
|---|---|---|
| $K = m$ | 0 | $> 1$ (bad) |
| $K = \alpha m$, $\alpha < 1$ | $\uparrow$ up | $\downarrow$ down |

# What did we learn from decision trees?

- Bias-Variance tradeoff formalized

$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{(k-1)\ln n + (2k-1)\ln(k+1) + \ln\frac{1}{\delta}}{2m}}$$

- Moral of the story:

  Complexity of learning not measured in terms of "size hypothesis space", but in maximum *number of points* that allows consistent classification

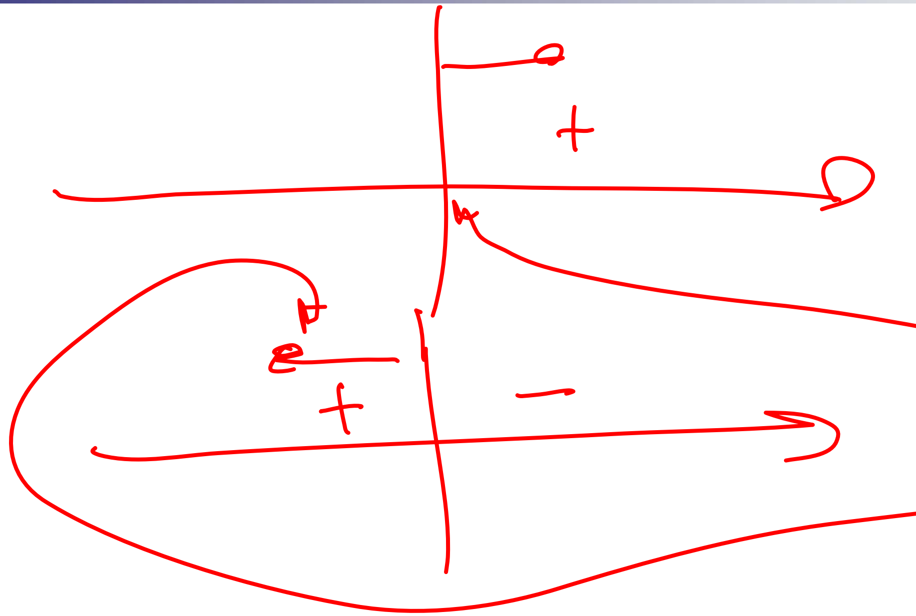
  - Complexity *m* – no bias, lots of variance
  - Lower than *m* – some bias, less variance

# What about continuous hypothesis spaces?

$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{\ln|H| + \ln\frac{1}{\delta}}{2m}}$$

- Continuous hypothesis space: linear classifiers..
  - |H| = ∞
  - Infinite variance???
- **As with decision trees, only care about the maximum number of points that can be classified exactly!**

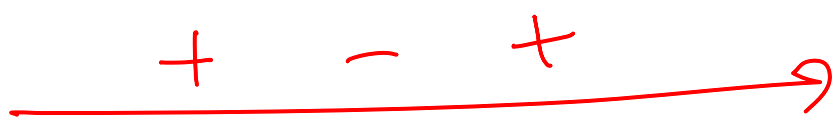
# How many points can a linear boundary classify exactly? (1-D)




2 points in 1-D

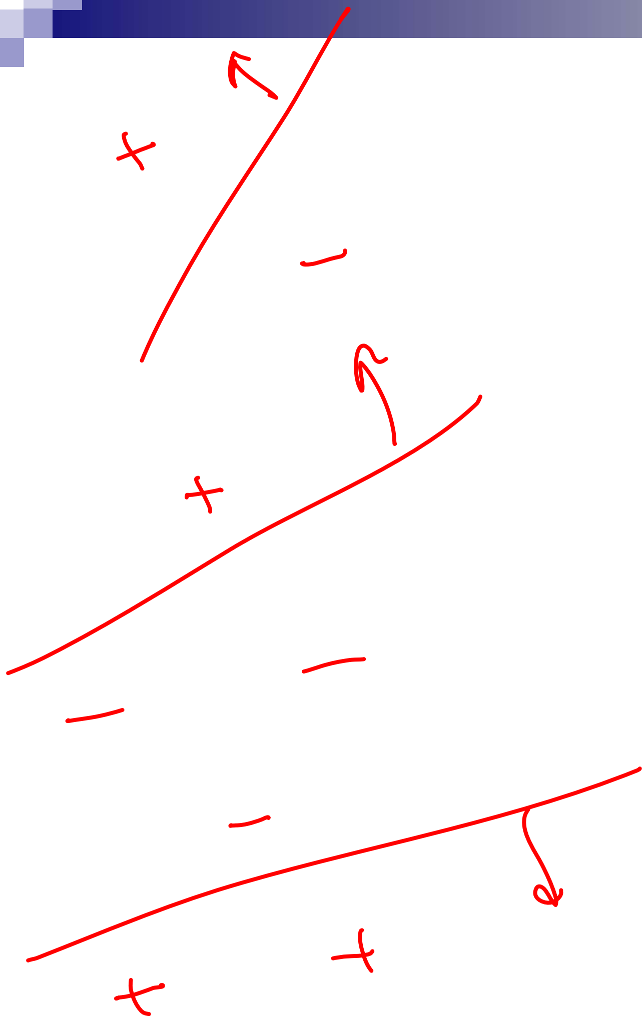
2 parameters!

- threshold
- direction




can't do it

# How many points can a linear boundary classify exactly? (2-D)




- you pick locations
- adversary picks labels

you

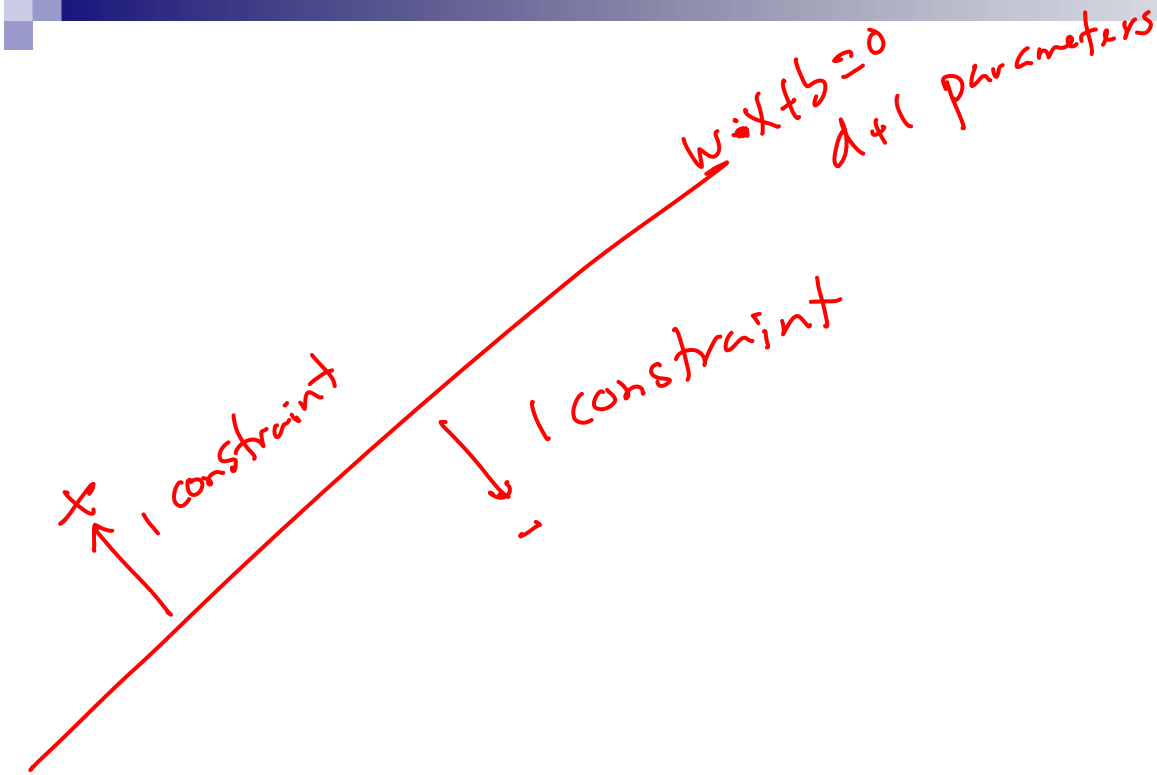
adv.

you pick

adversary: XOR

oops...

# How many points can a linear boundary classify exactly? (d-D)



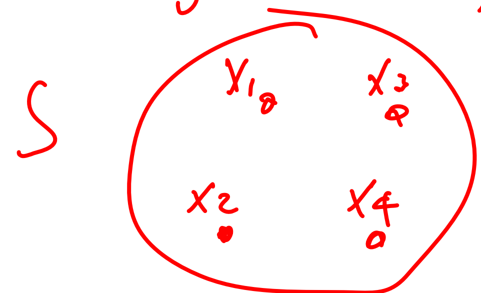

d+1 parameters

d+1 constraints

↳ d+1 datapoints

# Shattering a set of points

give me points




*Definition:* a **dichotomy** of a set $S$ is a partition of $S$ into two disjoint subsets.

dichotomy: set of labels

$S = S_1 \cup S_2$ $\quad S_1 \cap S_2 = \emptyset$

$S_1 \to +$

$S_2 \to -$

*Definition:* a set of instances $S$ is **shattered** by hypothesis space $H$ if and only if for every dichotomy of $S$ there exists some hypothesis in $H$ consistent with this dichotomy.

$\forall$ dichotomies

$\exists$ h labels correctly:

$\{x_1, x_2\} \to +$ $\quad \{x_3, x_4\} \to -$ $\}$ choose $h_7$

$\{x_3\} \to +$ $\quad \{x_1, x_2, x_4\} \to -$ $\}$ choose $h_{52}$

$\forall$ splits $\exists$ h $\to$ shattered !!

# VC dimension

*Definition:* The **Vapnik-Chervonenkis dimension**, $VC(H)$, of hypothesis space $H$ defined over instance space $X$ is the size of the largest finite subset of $X$ shattered by $H$. If arbitrarily large finite sets of $X$ can be shattered by $H$, then $VC(H) \equiv \infty$.

hyperplane in 2d

→ can't shatter

but I don't care...

I get to pick locations

I pick:

# PAC bound using VC dimension

- **Number of training points that can be classified exactly is VC dimension!!!**
  - **Measures relevant size of hypothesis space, as with decision trees with k leaves**
  - **Bound for infinite dimension hypothesis spaces:**

$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{VC(H)\left(\ln \frac{2m}{VC(H)} + 1\right) + \ln \frac{4}{\delta}}{m}}$$

| | | |
|---|---|---|
| high VC dim. | ↓ | ↑ |
| low VC dim. | ↑ | ↓ |

# Examples of VC dimension

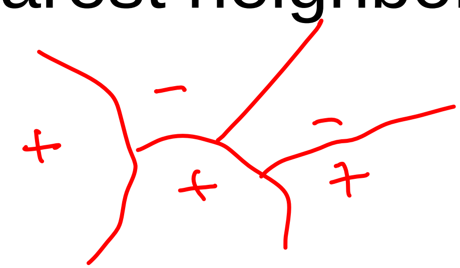
$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{VC(H)\left(\ln \frac{2m}{VC(H)} + 1\right) + \ln \frac{4}{\delta}}{m}}$$

- Linear classifiers:
  - VC(H) = d+1, for *d* features plus constant term *b*
- Neural networks
  - VC(H) = #parameters
  - Local minima means NNs will probably not find best parameters
- 1-Nearest neighbor?

(In this def., an input point is its own NN.)

$\downarrow$

VC = ∞



# Another VC dim. example

- What's the VC dim. of decision stumps in 2d?

VC at least 3

$\boxed{X_1}$: $x_1 \geq t$ → T/F, $x_1 < t$ → T/F

$\boxed{X_2}$: $x_2 \geq t$ → T/F, $x_2 < t$ → T/F

$x_2 \geq t$

prove ∃ no 4 points that can be labeled correctly

$(x_1, x_2, x_3, x_4)$

$x_1$ $\min_{xy}$ coord. → +

$x_2$ max $xy$ → +

$x_3, x_4$ → −

∃ not $t$ : $x_1 \geq t$ or $x_2 \geq t$ obtains these labels

# PAC bound for SVMs

- SVMs use a linear classifier
  - For *d* features, VC(H) = d+1:

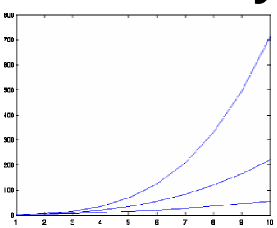
$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{(d+1)\left(\ln\frac{2m}{d+1}+1\right)+\ln\frac{4}{\delta}}{m}}$$

# VC dimension and SVMs: Problems!!!

**Doesn't take margin into account**

$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{(d+1)\left(\ln \frac{2m}{d+1} + 1\right) + \ln \frac{4}{\delta}}{m}}$$

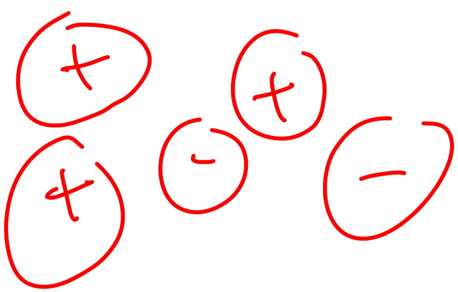
- What about kernels?
  - Polynomials: num. features grows really fast = Bad bound



$$\text{num. terms} = \binom{p+n-1}{p} = \frac{(p+n-1)!}{p!(n-1)!}$$

n – input features
p – degree of polynomial

  - Gaussian kernels can classify any set of points exactly

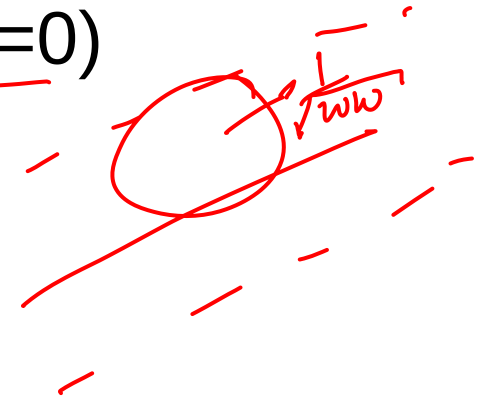


bound bad!!

"VC = ∞"

# Margin-based VC dimension

- H: Class of linear classifiers: $\mathbf{w}.\Phi(\mathbf{x})$ (b=0)
  - training example
  - Canonical form: $\min_j |\mathbf{w}.\Phi(\mathbf{x}_j)| = 1$
- $VC(H) = R^2\, \mathbf{w}.\mathbf{w}$ $= \frac{R^2}{\text{margin}^2}$
  - Doesn't depend on number of features!!!
  - $R^2 = \max_j \Phi(\mathbf{x}_j).\Phi(\mathbf{x}_j)$ – magnitude of data
  - $R^2$ is bounded even for Gaussian kernels $\rightarrow$ bounded VC dimension




- Large margin, low $\mathbf{w}.\mathbf{w}$, low VC dimension – Very cool!

# Applying margin VC to SVMs?

$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \sqrt{\frac{VC(H)\left(\ln \frac{2m}{VC(H)} + 1\right) + \ln \frac{4}{\delta}}{m}}$$

- VC(H) = $R^2$ **w.w** $\beta$
  - $R^2 = \max_j \Phi(\mathbf{x}_j).\Phi(\mathbf{x}_j)$ – magnitude of data, doesn't depend on choice of **w**
- SVMs minimize **w.w**

Smallest VC dim. hyp. consistent with data

- **SVMs minimize VC dimension to get best bound?**
- **Not quite right: ☹**
  - **Bound assumes VC dimension chosen before looking at data**
  - **Would require union bound over infinite number of possible VC dimensions…**
  - **But, it can be fixed!**

# Structural risk minimization theorem

$$\text{error}_{true}(h) \le \text{error}^{\gamma}_{train}(h) + C\sqrt{\frac{\frac{R^2}{\gamma^2}\ln m + \ln\frac{1}{\delta}}{m}}$$

simplest
Structure
→ minimum
error.

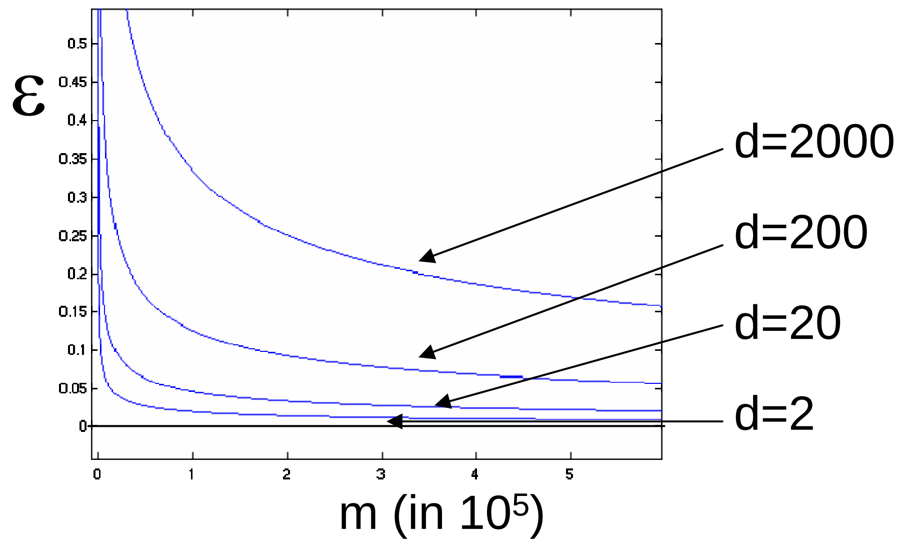
$$\text{error}^{\gamma}_{train}(h) = \text{num. points with margin} < \gamma$$

fix margin $\gamma$
how much of data do I get right.

- For a family of hyperplanes with margin $\gamma>0$
  - **w.w** $\leq 1$
- SVMs maximize margin $\gamma$ + hinge loss
  - Optimize tradeoff training error (bias) versus margin $\gamma$ (variance)

# Reality check – Bounds are loose

$$\text{error}_{true}(h) \leq \text{error}_{train}(h) + \underbrace{\sqrt{\frac{(d+1)\left(\ln \frac{2m}{d+1} + 1\right) + \ln \frac{4}{\delta}}{m}}}_{\epsilon}$$




- Bound can be very loose, why should you care?
  - There are tighter, albeit more complicated, bounds
  - Bounds gives us formal guarantees that empirical studies can't provide
  - Bounds give us intuition about complexity of problems and convergence rate of algorithms

# What you need to know

- Finite hypothesis space
  - Derive results
  - Counting number of hypothesis
  - Mistakes on Training data
- Complexity of the classifier depends on number of points that can be classified exactly
  - Finite case – decision trees
  - Infinite case – VC dimension
- Bias-Variance tradeoff in learning theory
- Margin-based bound for SVM
- Remember: will your algorithm find best classifier?

# Big Picture

Machine Learning – 10701/15781
Carlos Guestrin
Carnegie Mellon University

March 6th, 2006

# What you have learned thus far

- Learning is function approximation
- Point estimation
- Regression
- Naïve Bayes
- Logistic regression
- Bias-Variance tradeoff
- Neural nets
- Decision trees
- Cross validation
- Boosting
- Instance-based learning
- SVMs
- Kernel trick
- PAC learning
- VC dimension
- Margin bounds
- Mistake bounds

# Review material in terms of…

- Types of learning problems
- Hypothesis spaces
- Loss functions
- Optimization algorithms

# Text Classification


Company home page

vs

Personal home page

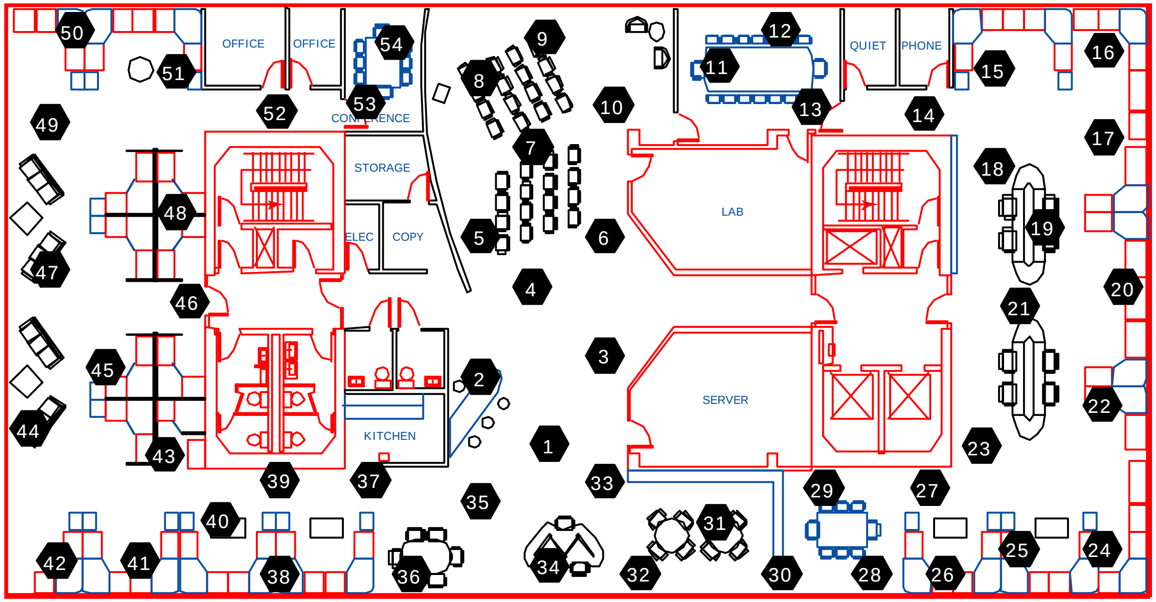
vs

Univeristy home page

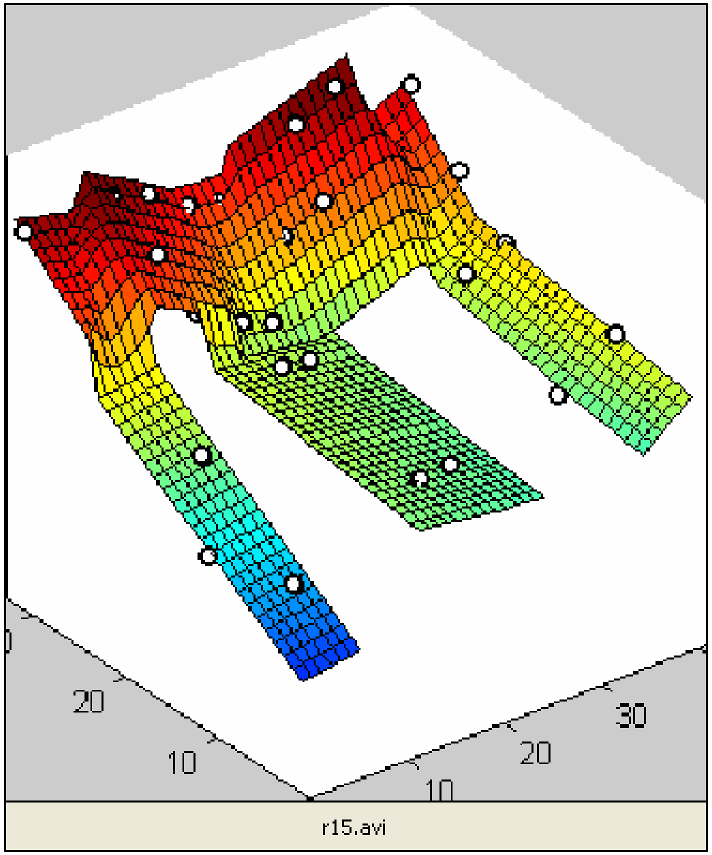
vs

…

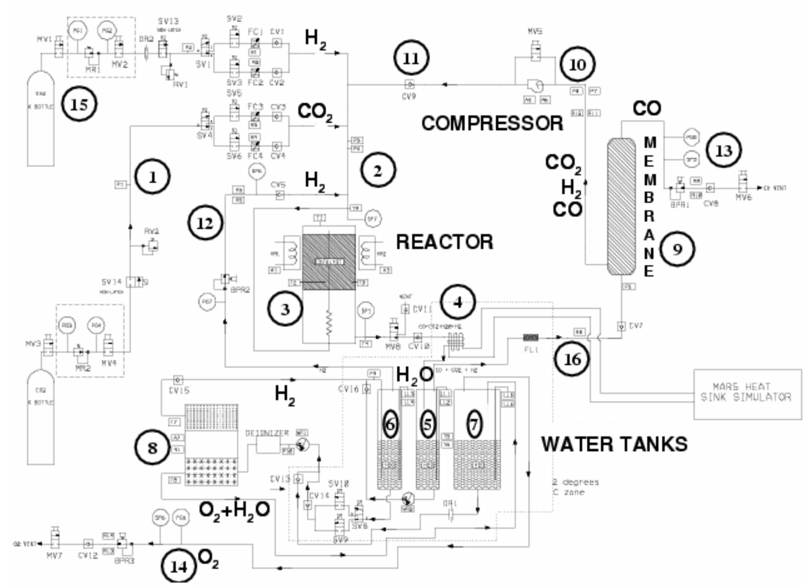
# Function fitting




Temperature data



# Monitoring a complex system

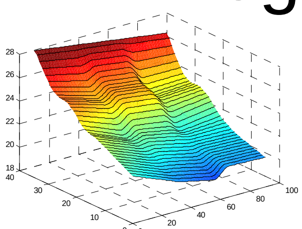



- Reverse water gas shift system (RWGS)
- Learn model of system from data
- Use model to predict behavior and detect faults

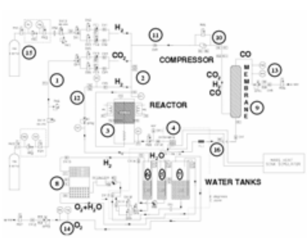
# Types of learning problems

- Classification


- Regression



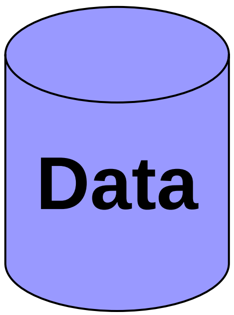

- Density estimation




**Input – Features**

**Output?**

# The learning problem




$<x_1,\ldots,x_n,y>$

**Learning task**

**Features/Function approximator**

**Loss function**

**Optimization algorithm**

**Learned function**

# Comparing learning algorithms

- Hypothesis space
- Loss function
- Optimization algorithm

# Naïve Bayes versus Logistic regression

**Naïve Bayes**

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

$$P(X|Y) = \prod_i P(X_i|Y)$$

**Logistic regression**

$$P(Y=1|x) = \frac{1}{1 + exp(w_0 + \sum_i w_i x_i)}$$

# Naïve Bayes versus Logistic regression – Classification as density estimation

$$P(Y|X)$$

- Choose class with highest probability

- In addition to class, we get certainty measure

# Logistic regression versus Boosting

## Logistic regression

$$P(Y = y_i|\mathbf{x}) = \frac{1}{1 + exp(-y_i(\mathbf{w}.\mathbf{x} + b))}$$

Log-loss

$$\sum_{j=1}^{m} \log \left[1 + exp(-y_i(\mathbf{w}.\mathbf{x}_j + b))\right]$$

## Boosting

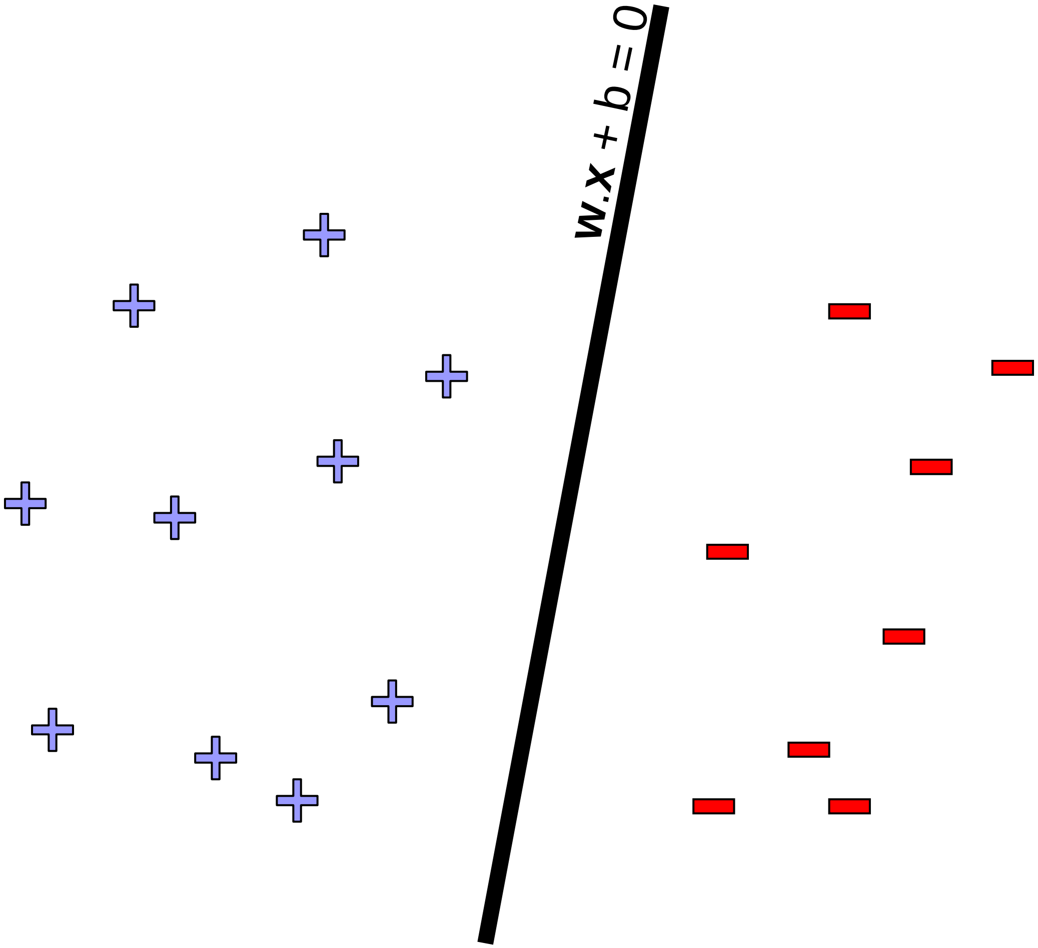
Classifier

$$sign\left(\sum_{t=1}^{T} \alpha_t h_t(\mathbf{x})\right)$$

Exponential-loss

$$\frac{1}{m}\sum_{j=1}^{m} \exp\left(-y_j \sum_{t=1}^{T} \alpha_t h_t(\mathbf{x_j})\right)$$

# Linear classifiers – Logistic regression versus SVMs



# What's the difference between SVMs and Logistic Regression? (Revisited again)

| | **SVMs** | **Logistic Regression** |
|---|---|---|
| **Loss function** | Hinge loss | Log-loss |
| **High dimensional features with kernels** | Yes! | Yes! |
| **Solution sparse** | Often yes! | Almost always no! |
| **Type of learning** | | |

# SVMs and instance-based learning

$$\mathbf{w} \cdot \Phi(\mathbf{x}) = \sum_i \alpha_i y_i K(\mathbf{x}, \mathbf{x}_i)$$

$$b = y_k - \sum_i \alpha_i y_i K(\mathbf{x}_k, \mathbf{x}_i)$$

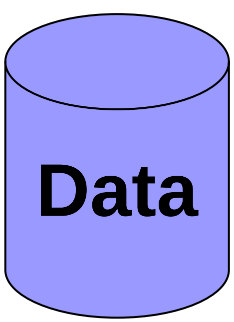
for any $k$ where $C > \alpha_k > 0$

Classify as

## SVMs

$$sign\left(\sum_i \alpha_i y_i K(\mathbf{x}, \mathbf{x}_i) + b\right)$$

## Instance based learning




<x1,…,xn,y>

Classify as

$$P(y \mid \mathbf{x}) = \frac{\sum_i y_i K(\mathbf{x}, \mathbf{x}_i)}{\sum_i K(\mathbf{x}, \mathbf{x}_i)} > 0.5?$$

$$sign\left(\sum_i y_i K(\mathbf{x}, \mathbf{x}_i) - 0.5 \sum_i K(\mathbf{x}, \mathbf{x}_i)\right)$$

# Instance-based learning versus Decision trees

**1-Nearest neighbor** | **Decision trees**

# Logistic regression versus Neural nets

$$g(w_0 + \sum_i w_i x_i) = \frac{1}{1 + e^{-(w_0 + \sum_i w_i x_i)}}$$

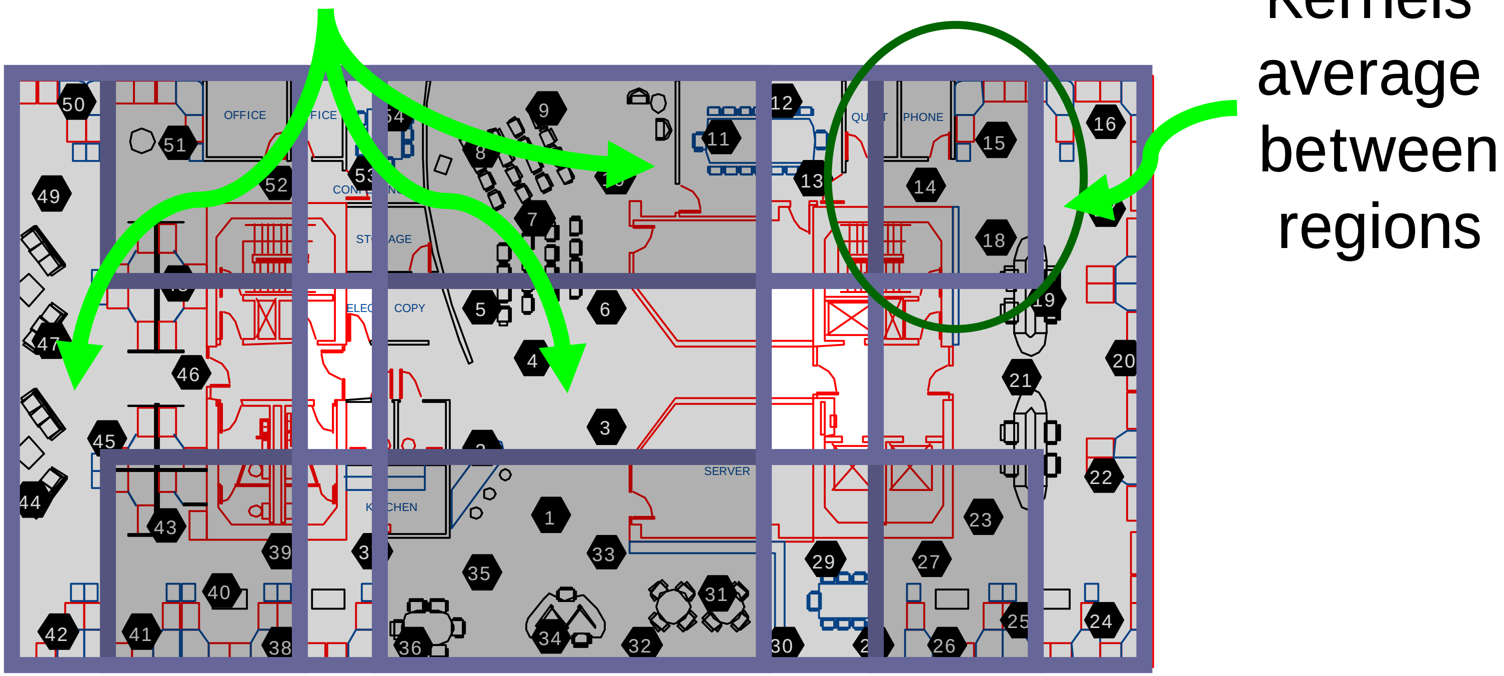
| **Logistic regression** | **Neural Nets** |
| --- | --- |

# Linear regression versus Kernel regression

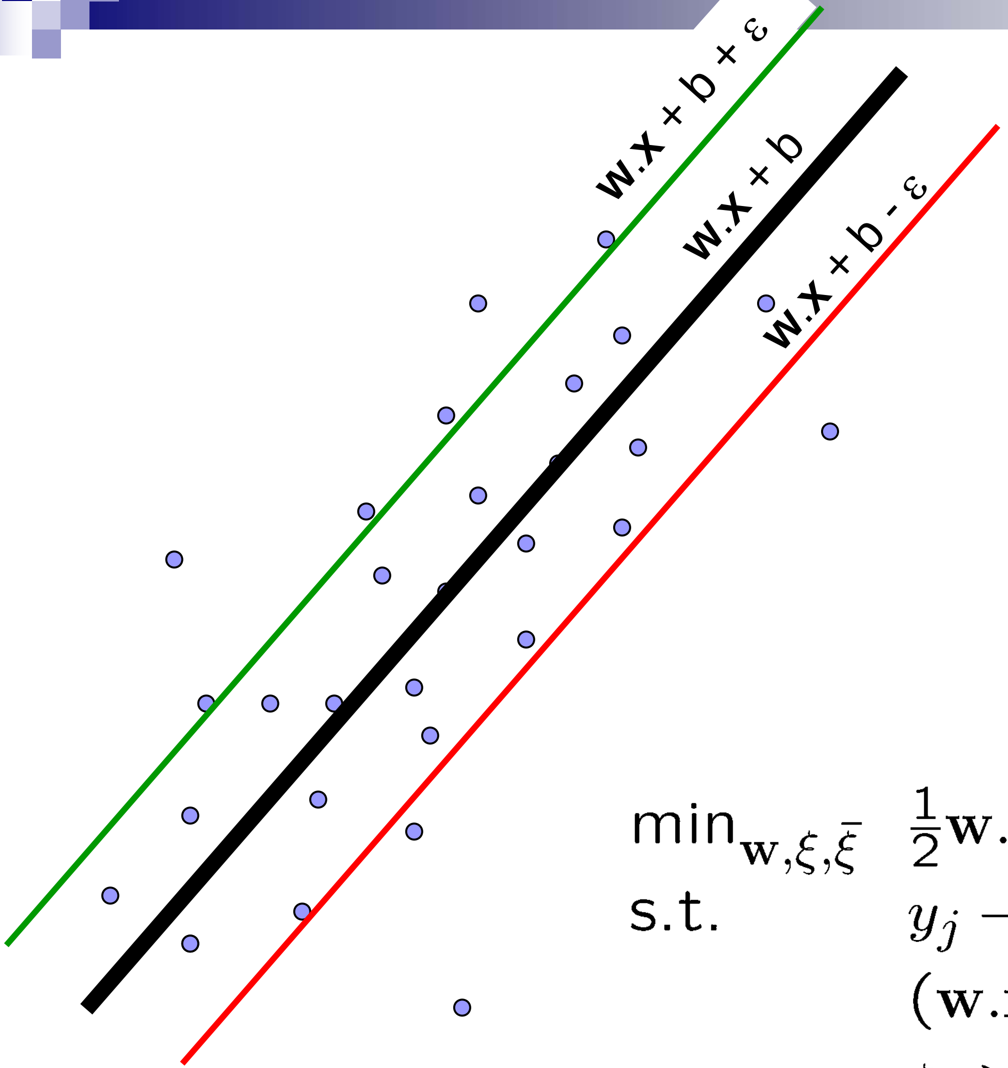
| **Linear Regression** | **Kernel regression** | **Kernel-weighted linear regression** |
|---|---|---|

# Kernel-weighted linear regression

- Local basis functions for each region



# SVM regression




$$\min_{\mathbf{w},\xi,\bar{\xi}} \quad \frac{1}{2}\mathbf{w}.\mathbf{w} + C\sum_{j=1}^{m}(\xi_j + \bar{\xi}_j)$$

$$\text{s.t.} \quad y_j - (\mathbf{w}.\mathbf{x}_j + b) \leq \epsilon + \xi_j$$

$$(\mathbf{w}.\mathbf{x}_j + b) - y_j \leq \epsilon + \bar{\xi}_j$$

$$\xi_j \geq 0, \quad \bar{\xi}_j \geq 0, \quad \forall j$$

# BIG PICTURE

## (a few points of comparison)

learning task

loss function

| | |
|---|---|
| DE | density estimation |
| Cl | Classification |
| Reg | Regression |
| LL | Log-loss/MLE |
| Mrg | Margin-based |
| RMS | Squared error |

Boosting
Cl, exp-loss

Naïve Bayes
DE, LL

SVM regression
Reg, Mrg

Logistic regression
DE, LL

SVMs
Cl, Mrg

kernel regression
Reg, RMS

Instance-based Learning
DE,Cl,Reg

Neural Nets
DE,Cl,Reg,RMS

Decision trees
DE,Cl,Reg

linear regression
Reg, RMS

**This is a very incomplete view!!!**