convex optimization problem

\[ \min \ f(x) \]
\[ \text{st. } g_i(x) \leq 0 \quad i = 1, \ldots, m \]
\[ h_j(x) = 0 \quad j = 1, \ldots, n \]

\( f \) convex.
\( g_i \) convex.
\( h_j \) linear

\[ h_j(x) = a_j^T x + b_j \]

\[ a_j^T x + b_j \leq 0 \quad d_j(x) = 0 \]
\[ -a_j^T x - b_j \leq 0 \]
\( f(x^*) \leq f(y) \quad \forall \ y \text{ st.} \quad \|x^*-y\| \leq R \)

Note: \( x^* \) must be globally optimal. Why?

Suppose \( z \) is feasible

and \( f(z) \leq f(x^*) \)

\[
\tag{R}
\|x^*-z\| > R
\]

\[\alpha = \frac{R}{\|x^*-z\|} \quad y = (1-\alpha)x^* + \alpha z\]

\[
\|x^*-y\| = \|x^* - z + \alpha (x^*-z)\|
\]

\[
= \|\alpha (x^*-z)\|
\]

\[
= \alpha \|x^*-z\| = R
\]

\[
\frac{f(y)}{f(z)} = f\left((1-\alpha)x^* + \alpha z\right)
\]

\[
\leq \alpha f(x^*) + \alpha f(z)
\]

\[
< (1-\alpha) f(x^*) + \alpha f(x^*)
\]

\[
= f(x^*).
\]

Contradiction
problems
know how
to solve
in stats/ML

some rare
gems in there!

Convex

First-order
methods

dastic, useful
algos

Second-order
methods

when to use
what?