Tips for written homework

- For HW1, you’ll need to submit the written part through Gradescope.
- If you’re registered, please access Gradescope via Canvas: open Canvas, select Gradescope from the left navigation column.
  - this lets us link your Gradescope account to Canvas so you get credit.
  - if you’re not registered, should still be able to access Gradescope directly.
- Two options for preparing submission:
  - Handwrite and scan.
  - Type using a markup language.
Handwrite and scan

- Handwrite legibly!
- Use a scanning app on your phone (see suggestions on course website)
  - don’t just take a photo; this will result in skewed, poor-contrast, badly cropped images that are hard for the TAs to work with
- Upload PDF to Gradescope
### Markup languages

- Two most common are LaTeX and Markdown; both work well but need setup
  - Both can produce PDFs to submit on Gradescope
- For lecture notes, I use the VSCode editor (Markdown is built in) with Markdown+Math (enables LaTeX math) and Markdown Extended (better CSS)

#### Complete spaces

Above we described how to think of matrices or functions as vectors in a vector space. We also described how to upgrade a vector space to an inner product space by defining an inner product $\langle x, y \rangle$. For example,

- A useful inner product for matrices is
  $$
  \langle X, Y \rangle = \sum_{i=1, j=1}^{i=m, j=n} X_{ij}Y_{ij} = \text{tr}(X^TY) = \text{tr}(YX^T)
  $$
\[ \left\{ \begin{array}{l} w_1, w_2 \in \mathbb{R}^3 \\ \text{span} \\ \left( \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right) \end{array} \right. \]

Orthogonal case 

\[ \langle x, y \rangle = u \cdot v \]
Exercise: equivalent vector spaces

• Are these vector spaces the same? $\mathbb{R}^{mx1}$ $\mathbb{R}^{1xm}$ $\mathbb{R}^m$
  ▶ A: yes, they’re the same
  ▶ B: no, they’re different
  ▶ C: they’re different but equivalent
Exercise: basis

• Consider the vector space of degree-2 polynomials in a real variable x with the basis 1, x, 2x^2 – 1 (first three Chebyshev polynomials)

• What is the representation of x^2 in this basis?

\[ \frac{1}{2} \cdot 1 + 0 + \frac{1}{2} (2x^2 - 1) \rightarrow \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \]

• What is the representation of (x-1)^2?
Let \( \mathbf{w} \in \mathbb{R}^2 \), we define the decision surface as:

\[
\mathbf{x} \text{ if } \mathbf{w} \cdot \mathbf{x} > b
\]

\[
0 \text{ if } \mathbf{w} \cdot \mathbf{x} = b
\]

\[
0 \text{ if } \mathbf{w} \cdot \mathbf{x} < b
\]

Given \( \mathbf{x} \in \mathbb{R}^2 \) and \( \mathbf{w} \) as the weight vector, we can formulate the decision function as:

\[
\sum_{i=1}^{n} \mathbf{x}_i \cdot \mathbf{w} = 0
\]

The decision function is equal to zero if the point is on the decision surface.

\[
f(x) = \mathbf{w} \cdot \mathbf{x} - b
\]

The margin of the decision surface can be calculated using the distance from the origin to the decision surface.

\[
\text{margin} = \frac{b}{\|\mathbf{w}\|}
\]
Feature transforms

\[ (x_1) \mapsto \begin{bmatrix} x_1 \\ x_2 \\ x_1 x_2 \\ x_1^2 \\ x_2^2 \end{bmatrix} \]

\[ \omega \cdot \phi(x) - b \\ \omega_1 x_1 + \omega_2 x_2 + \omega_3 x_1 x_2 + \omega_4 x_1^2 + \omega_5 x_2^2 - b = 0 \]
Feature transforms

\[ (x_1, x_2) \rightarrow (x_1') \]

- Feature transform diagram with points before and after transformation.
\( l(\mathbf{b}_i) \in V \)

\( l(\mathbf{b}_0) \in V \)

\( l(\mathbf{b}_i) = l_{ij} c_i + l_{2j} c_2 + \ldots + l_{mj} c_m \)
\[ f(((x, y)) = \begin{pmatrix} x + y \\ 2x + 2y \end{pmatrix} \quad \rightarrow \text{range} = \text{span} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \]

\[ \rightarrow \text{null} = \text{span} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \]

\[ \text{rank} + \text{nullity} = n \]

**Adjoint**

\[ f^* \langle (u, v), f((x, y)) \rangle = \langle f^*((u, v)), (x, y) \rangle \]

\[ (-2)^2 \in \text{null}(f^*) \quad f^* ((u, v)) = (u + 2v) \]

**Inverse**

\[ (y) = f(((x, y))) \]

\[ f^{-1}((u, v)) = (x, y) \]

\[ \text{range}(f) \perp \text{null}(f) \]
Exercise: Gaussian elimination

• Suppose
  ▶ $x + y + z = 3$
  ▶ $2x + y = 5$
  ▶ $-x + y - 2z = 4$
• What are $x$, $y$, $z$?
\[
\begin{bmatrix}
1 & 1 & 1 \\
2 & 0 & 1 \\
1 & -2 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
3 \\
1 \\
1
\end{bmatrix}
\]

\[\begin{align*}
2x + y - 2z &= 5 \\
x + y - 2z &= 4 \\
0 + 2y - 2z &= 7 \\
0 + y - 2z &= -1 \\
0 + 0 - 5z &= 5
\end{align*}\]