10-601 Machine Learning, Fall 2009: Homework 2

Due: Wednesday, September 16nd, 10:30 am

Instructions There are 4 questions on this assignment worth the total of 100 points. Please hand in a hard copy at the beginning of the class. Refer to the webpage for policies regarding collaboration, due dates, and extensions.

1 Bayesian networks and factor graphs [12 pts]

1. For each of the networks given in Figure 1 (a,b,c), do the following statements hold? Please explain your reasoning.
   - $A \perp C \mid B, D$ [3 x 2 pts]
   - $B \perp D \mid A, C$ [3 x 2 pts]

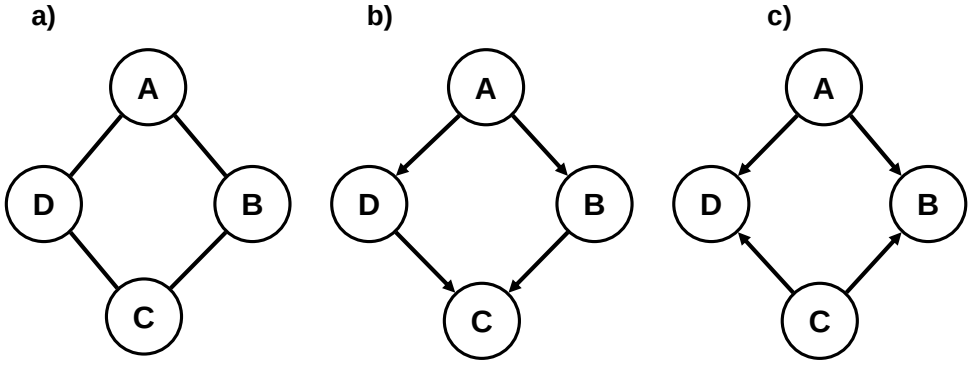

Figure 1: Factor graph (a) and Bayesian networks(b,c) for Problem 1.

2 Conditional probabilities [8 pts]

Prove or disprove (by providing a counter-example) each of the following properties of independence:

1. $\mathbf{P}(X, Y \mid Z) = \mathbf{P}(X \mid Y, Z)\,\mathbf{P}(Y \mid Z)$ [3 pts]
2. $(X \perp Y \mid Z)$ and $((X, Y) \perp W \mid Z)$ implies $(X \perp W \mid Z)$ (This statement means: If X is independent of Y given Z and the joint probability distribution of X and Y is independent of W given Z, then X and W are independent given Z.) [5 pts]

3 Disease and symptoms [20 pts]

A patient goes to the doctor for a medical condition, the doctor suspects three diseases as the cause of the condition. The three diseases are D_1, D_2, D_3, which are marginally independent from each other. There are four symptoms S_1, S_2, S_3, S_4 which the doctor wants to check for presence in order to find the most probable cause of the condition. The symptoms are conditionally dependent to the three diseases as follows: S_1 depends only on D_1, S_2 depends on D_1 and D_2. S_3 is depends on D_1 and D_3, whereas S_4 depends only on D_3. Assume all random variables are Boolean, they are either 'true' or 'false'.

1. Draw the Bayesian network for this problem. [2 pts]
2. Write down the expression for the joint probability distribution as a product of conditional probabilities. [2 pts]
3. What is the number of independent parameters that is required to describe this joint distribution? [3 pts]
4. Assume there were no conditional independence between the variables, how many independent parameters would be required then? [3 pts]
5. What is the Markov Blanket of variable S_2? [2 pts]
6. What is an example of the 'explaining away' phenomenon in the graph? [2 pts]
7. If we observe the fourth symptom,$(S_4 = true)$, for which diseases do we gain information? [3 pts]
8. Suppose we observed second symptom is present in the patient $(S_2 = true)$, what does observing the fourth symptom $(S_4 = true)$ tell us now? [3 pts]

4 Bayesian about Happiness [60 points]

As part of a comprehensive study of the role of 10-601 on people's happiness we have been collecting important data from graduating students. In an entirely optional survey that all students are required to complete, we ask the following highly objective questions:

- Do you party frequently [`Party`: Yes/No]?
- Are you wicked smart [`Smart`: Yes/No]?
- Are you creative [`Creative`: Yes/No]? (Please only answer Yes or No)
- Did you do well on all your homework assignments? [`HW`: Yes/No]
- Do you use a Mac? [`Mac`: Yes/No]
- Did your 10-601 project succeed? [`Project`: Yes/No]
- Did you succeed in your most important class (which is 10-601)? [`Success`: Yes/No]
- Are you currently Happy? [`Happy`: Yes/No]

You can obtain the comma-separated survey results from http://www.cs.cmu.edu/~ggordon/10601/hws/hw2/students.csv.zip. Each row in `students.csv` corresponds to the responses of a separate student. The columns in `students.csv` correspond to each question (random variable) in the order `Party`, `Smart`, `Creative`, `HW`, `Mac`, `Project`, `Success`, and `Happy`. The entries are either zero, corresponding to `No` response, or one, corresponding to a `Yes` response. After consulting a behavioral psychologist we obtained the following complete set of conditional relationships:

- `HW` depends only on `Party` and `Smart`
- `Mac` depends only on `Smart` and `Creative`
- `Project` depends only on `Smart` and `Creative`
- `Success` depends only on `HW` and `Project`
- `Happy` depends only on `Party`, `Mac`, and `Success`

4.1 Understanding The Model [12 Points]

1. Draw the Bayesian network.
2. Write joint distribution as a product of conditional probabilities.
3. What is the number of independent parameters needed for each conditional probability table?
4. What is the total number of independent parameters?

4.2 D-Separation [8 Points]

Using only the Bayesian network structure from part 4.1, answer the following True/False questions and provide a brief explanation:

1. `Party` is independent of `Success` given `HW`.
2. `Party` is independent of `Smart` given `Success`.
3. `Party` is independent of `Creative` given `Happy`.

4.3 Confounded Intelligence [10 Points]

1. Using only the data in `students.csv` and Matlab calculate the correlation between success on the homework `HW` and success on the project `Project`. You do not need to use the Bayesian network for this question. (Hint: Consider using the `cov` function in Matlab.)
2. From the model structure, identify a potential common cause variable which may explain the correlation between `HW` and `Project`.

4.4 Counting [15 Points]

Use Matlab and `students.csv` to calculate the parameters for each conditional probability table by counting with Laplace smoothing. Please consider formatting your conditional probability tables as shown in Table 1.

A	B	C	$\mathbf{P}(X = 1 \mid A, B, C)$
T	T	T	0.4
T	T	F	0.8
T	F	T	0.15
T	F	F	0.16
F	T	T	0.23
F	T	F	0.42
F	F	T	0.4
F	F	F	0.8

Table 1: An example conditional probability table for $\mathbf{P}(X \mid A, B, C)$.

4.5 Inference [15 Points]

Using any of the following software,

- **Recommended:** AISpace Graphical Tool http://www.aispace.org/bayes/version5.1.6/bayes.jnlp other formats (jar, exe, applet) are available http://www.aispace.org/downloads.shtml
- The Matlab Bayes Net Toolbox: http://people.cs.ubc.ca/~murphyk/Software/BNT/bnt.html
- WinBUGS (Bayesian Inference Using Gibbs Sampling) http://www.mrc-bsu.cam.ac.uk/bugs/

along with your conditional probability table estimates, calculate the following probabilities:

- What is the probability of being happy?

- What is the probability of being happy given that you party often, are wicked smart, but not very creative?
- What is the probability of being happy given that you are wicked smart and very creative?
- What is the probability of being happy given you do not party, and do well on all your homework and class project?
- What is the probability of being happy given you own a mac?
- What is the probability that you party often given you are wicked smart?
- What is the probability that you party often given you are wicked smart and happy?