Digital Images

- How are they formed?
- How can they be represented?
Image Representation

- Hardware
  - Storage
  - Manipulation
- Human
  - Conceptual
  - Mathematical

Iconic Representation

- What you think of as an image, ...
  - Camera
  - X-Ray
  - CT
  - MRI
  - Ultrasound
  - 2D, 3D, ...
  - etc
Iconic Representation

- And what you might not

Range Image  
Corresponding Intensity Image

Images from CESAR lab at Oak Ridge National Laboratory. Sourced from the USF Range Image Database: http://marathon.csee.usf.edu/range/Database.html. Acknowledgement thereof requested with redistribution.

Functional Representation

- An Equation
  - Typically continuous
- Fit to the image data
  - Sometimes the entire image
  - Usually just a small piece of it
- Examples (Quadratic Surfaces):
  - Explicit: \( z = ax^2 + by^2 + cxy + dx + ey + f \)
  - Implicit: \( 0 = ax^2 + by^2 + cz^2 + dxy + eyz + fyz + gx + hy + iz + j \)
Linear Representation

- Unwind the image
  - “Raster-scan” it
- Entire image is now a vector
  - Now we can do matrix operations on it!
  - Often used in research papers

Probabilistic & Relational Representations

- Probability & Graphs
- Discussed later (if at all)

Spatial Frequency Representation

- Think “Fourier Transform”
- Multiple Dimensions!
- Varies greatly across different image regions
- High Freq. = Sharpness

- Steven Lehar’s details: http://sharp.bu.edu/~sllehary/fourier/fourier.html
Image Formation

- Sampling an analog signal
- Resolution
  - # Samples per dimension, OR
  - Smallest clearly discernable physical object
- Dynamic Range
  - # bits / pixel (quantization accuracy), OR
  - Range of measurable intensities
    - Physical meaning of min & max pixel values
    - light, density, etc.

Dynamic Range Example
(A slice from a Renal Angio CT: 8 bits, 4 bits, 3 bits, 2 bits)
An Aside: The Correspondence Problem

- My Definition:
  - Given two different images of the same (or similar) objects,
    for any point in one image determine the exact corresponding point in the other image
- Similar (identical?) to registration
- Quite possibly, it is THE problem in computer vision

Image Formation: Corruption

- There is an ideal image
  - It is what we are physically measuring
- No measuring device is perfect
  - Measuring introduces noise
    - \( g(x,y) = D(f(x,y)) \), where \( D \) is the distortion function
- Often, noise is additive and independent of the ideal image
Image Formation: Corruption

- Noise is usually not the only distortion
- If the other distortions are:
  - linear &
  - space-invariant
then they can *always* be represented with the convolution integral!
- Total corruption:
\[
g(x, y) = \iint_{-\infty}^{\infty} f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta + n(x, y)
\]

The image as a surface

- Intensity → height
  - In 2D case, but concepts extend to ND
- \( z = f(x, y) \)
- Describes a surface in space
  - Because only one \( z \) value for each \( x, y \) pair
  - Assume surface is continuous (interpolate pixels)
Isophote

- “Uniform brightness”
- $C = f(x, y)$
- A curve (2D) or surface (3D) in space
- Always perpendicular to image gradient
  - Why?

Isophotes & Gradient

- Isophotes are like contour lines on a topography (elevation) map.
- At any point, the gradient is always at a right angle to the isophote!
Ridges

- One definition:
  - Local maxima of the rate of change of gradient direction
  - Sound confusing?
  - Just think of ridge lines along a mountain
  - If you need it, look it up
  - Snyder references Maintz

Medial Axis

- Skeletal representation
- Defined for binary images
  - This includes segmented images!
- “Ridges in scale-space”
  - Details have to wait (ch. 9)

Image courtesy of Transcan/Data Europe
http://www.fcps.co.uk/motech.html
Neighborhoods

- Terminology
  - 4-connected vs. 8-connected
  - Side/Face-connected vs. vertex-connected
  - Maximally-connected vs. minimally-connected (ND)

- Connectivity paradox
  - Due to discretization

- Can define other neighborhoods
  - Adjacency not necessarily required

Curvature

- Compute curvature at every point in a (range) image
  - (Or on a segmented 3D surface)
- Based on differential geometry
- Formulas are in your book
- 2 scalar measures of curvature that are invariant to viewpoint, derived from the 2 principal curvatures, \( (K_1, K_2) \):
  - Mean curvature (arithmetic mean)
  - Gauss curvature (product)
    - \( =0 \) if either \( K_1=0 \) or \( K_2=0 \)