Lecture 15
Deformable / Non-Rigid Registration

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Registration:
“Rigid” vs. Deformable

- **Rigid Registration:**
  - Uses a simple transform, *uniformly* applied
  - Rotations, translations, etc.

- **Deformable Registration:**
  - Allows a non-uniform mapping between images
  - Measure and/or correct small, varying discrepancies by deforming one image to match the other
  - Usually only tractable for deformations of small spatial extent!
Deformable, i.e. Non-Rigid, Registration (NRR)

- Vector field (aka deformation field) $T$ is computed from A to B
- Inverse warp transforms B into A’s coordinate system
- Not only do we get correspondences, but...
- We also get shape differences (from $T$)

NRR Clinical Background

- Internal organs are non-rigid
- The body can change posture
  - Even skeletal arrangement can change
- Single-patient variations:
  - Normal
  - Pathological
  - Treatment-related
- Inter-subject mapping: People are different!
  - Atlas-based segmentation typically requires NRR
More Clinical Examples

- Physical brain deformation during neurosurgery
- Normal squishing, shifting and emptying of abdominal/pelvic organs and soft tissues
  - Digestion, excretion, heart-beat, breathing, etc.
- Lung motion during respiration can be huge!
- Patient motion during image scanning

Optical Flow

- Traditionally for determining motion in video—assumes 2 sequential images
- Detects small shifts of small intensity patterns from one image to the next
- Output is a vector field, one vector for each small image patch/intensity pattern
- Basic gradient-based formulation assumes intensity values are conserved over time
Optical Flow Assumptions

- Images are a function of space and time
- After short time $dt$, the image has moved $dx$
- Velocity vector $v = dx/dt$ is the optical flow

$$I(x, t) = I(x+dx, t+dt) = I(x+v\cdot dt, t+dt)$$

- Resulting optical flow constraint:

$$C_{of} = I_x \cdot v + I_t = 0$$

Optical Flow Constraint

- Optical flow constraint dictates that when an image patch is spatially shifted over time, that it will retain its intensity values
- Let image A = $I(x, t=0)$ and let B = $I(x, t=1)$
- Then $I_t = A(T) - B$

- This alone is not a sufficient constraint!
NRR Is Ill-Posed

- Review of well-posed problems:
  - A solution exists, is unique, and depends continuously on the data
  - Otherwise, a problem is ill-posed
- Ambiguity within homogenous regions:

Very Ill-Posed Problem

- NRR answer is not unique, and...
- NRR Search-space is often \( \infty \)-dimensional!
- Solution: Regularization
  - Adding a regularization term can provide provable uniqueness and a computable subspace
  - Regularization usually based on continuum mechanics
    - \( T \) is restricted to be physically admissible
    - We’re typically deforming physical anatomy, after all
    - Optimum \( T \) should deform “just enough” for alignment
NRR Regularization Methods

- Numerous continuum mechanical models available for regularization priors
  - Elastic
  - Diffusion
  - Viscous
  - Flow
  - Curvature
- Optimization is then physical simulation over time, \( t \), of trying to deform one image shape to match another
- This optimization has 3 equivalent formulations:
  - Global potential energy minimization
  - Variational or weak form, as used in finite-element methods
  - Euler-Lagrangian (E-L) equations, as used in finite-difference techniques

Langrangian View

- Elastic physical model:
  - How much have we stretched, etc., from our original image coordinates?
  - Simulation calculates the physical model’s resistance to deformation based on the total deformation from time \( t=0 \) to \( t=\text{now} \).
- \( T \) is the final vector field \( \tilde{u}_f \):
  \[
  \tilde{u}_f = \tilde{u}( t=t_{\text{final}} ) \\
  A( X + \tilde{u}_f ) \sim B(x) \\
  X = x - \tilde{u}_f 
  \]
Eulerian View

- Viscous-flow physical model:
  - How much have we flowed from our immediately previous simulation state?
  - Simulation calculates the physical model’s resistance to deformation based on the incremental deformation from time \( t=(\text{now}-1) \) to \( t=\text{now} \).
- \( T \) is the aggregate flow of \( x(t) \), based on accumulated optical flow (i.e. velocity) \( v(t) \):
  \[
  x(t) = x + v(t)
  \]
  \[
  A( x(=t_{\text{final}}) ) \sim B(x)
  \]

Deformation at time \( t \):

Deformation at time \( t + dt \):

Comparison of Regularization Reference Frames

- Langrangian
  - The entire deformation is regularized
  - Well constrained for “normal” physical deformation
  - Too constrained to achieve “large” deformations
  - Not ideal for many inter-subject mapping tasks
- Eulerian
  - Only the incremental updates are regularized
  - Underconstrained for “normal” physical deformation
  - Readily achieves large, inter-subject deformations
  - Unrealistic transformations can result
Transient Quadratic (TQ) Approach

- Enables better-constrained large deformations

- Uses Lagrangian regularization for specified time interval, followed by a re-gridding strategy
  - After an interval’s deformation reaches a threshold, we begin a new interval for which the last deformation becomes the new starting point
  - TQ thus resets the coordinate system while permanently storing the past state of the algorithm

- Results in a hybrid E+L physical model, resembling soft, stretchable plastic
  - Maintains the elastic regularization for a given time then takes on a new shape until new stresses are applied

Optical Flow Regularized

\[ E_D(v) = \int_{\Omega} \Phi(C_{of}) d\Omega + \int_{\Omega} \Psi(v) d\Omega \]

- First term adjusts \( v \) to make the images match (wants \( C_{of} = 0 \) within the bounded domain \( \Omega \))

\[ \text{e.g., } \Phi(C_{of}) = C_{of}^2 \]

\[ \text{e.g., } \int_{\Omega} \Psi(v) d\Omega = \|Lv\|^2 \]

- Goal: Minimize global potential energy, \( E_D \)
- Second term adds a stabilizing function \( \Psi \), typically a regulator operator \( L \) applied to \( v \)
Optical Flow E-L Regularized

- After deriving the E-L equations & setting their derivative = 0, we find that the...
- Potential energy minimum will occur when:
  \[ I_x (I_x \cdot v + I_t) - v_{xx} = 0 \]

- First term minimizes optical flow constraint
- Second term minimizes Laplacian (i.e. roughness) of velocity field \( v \)
- Note that this equation is evaluated *locally*
  - Allows for efficient implementation

Demons Algorithm: Math

- *Very* efficient gradient-descent NRR algorithm
- Originally conceived as having “demons” push image level sets around, but is also...
- Based on E-L regularized optical flow
- Alternates between minimizing each half of the previous equation:
  - Descent in optical flow direction, based on:
    \[ I_x (I_x \cdot v + I_t) = 0 \]
  - Smoothing, which estimates \( v_{xx}=0 \) with a difference-of-Gaussian filter, by applying a Gaussian on each iteration
Demons Algorithm: Code

- Initialize solution (i.e. total vector field) = Identity
- Loop:
  - Estimate vector field update
    - Use (stabilized) optical flow
  - Add update to total vector field
  - Blur total vector field (for regularization)

- Allows much larger deformation fields than optical flow alone.
- **Langrangian registration**: blur the total vector field (as above)
- **Eulerian registration**: blur the individual vector-field updates

Choices & Details

- There are many more NRR algorithms available
- Almost all of them are slower than demons, but they may give you better results
- See the text for details, and lots of helpful pictures