So what’s the difference between a session type and an ordinary type anyway?

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Thirty Years of Session Types
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Apologies for impressionistic style and lack of references
What’s not really different?

- Ordinary: data type vs. phrase type
- Session: message type vs. behavioral type
- Ordinary: intuitionistic propositions as simple types
- Session: linear propositions as session types
- Ordinary: preservation and progress
- Session: session fidelity and deadlock freedom
So what is special?

1. Integration of **global** and **local** types
   - Global types ∼ specifications
   - Local types ∼ implementations

2. **Substructural** (linear or affine) types
   - Reflect process state
   - Channel types evolve during communication

This talk focuses on **2**

What have we learned more broadly?
Example: a Store (or Network)

\[
\text{store}_A = \& \{ \ 	ext{ins} : A \multimap \text{store}_A, \\
\quad \text{del} : \bigoplus \{ \ \text{none} : 1, \text{some} : A \otimes \text{store}_A \ \} \ \}\]

- Typing judgment for processes \( \Delta \vdash P :: (x : A) \)
  - Process \( P \) provides channel \( x \) of type \( A \)
  - \( P \) is client to channels in \( \Delta = (x_1 : A_1, \ldots, x_n : A_n) \)

- In linear logic / process calculus

<table>
<thead>
<tr>
<th>prop/type</th>
<th>provider action</th>
<th>continuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \multimap B )</td>
<td>external choice</td>
<td>receive choice</td>
</tr>
<tr>
<td>( A \rightarrow B )</td>
<td>implication</td>
<td>receive channel ( a : A )</td>
</tr>
<tr>
<td>( A \oplus B )</td>
<td>internal choice</td>
<td>send choice</td>
</tr>
<tr>
<td>( A \otimes B )</td>
<td>conjunction</td>
<td>send channel ( a : A )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>unit</td>
<td>send unit</td>
</tr>
</tbody>
</table>
store_A = &{\ ins : A \to store_A,
\ del : \oplus\{ none : 1, some : A \otimes store_A \} }

server :: (s : store_A) =
 recv s (\ ins \Rightarrow % s : A \to store_A
 recv s (x \Rightarrow % s : store_A
 ...)
 | del \Rightarrow % s : \oplus\{ none : 1, some : A \otimes store_A \}
 send s some ; % s : A \otimes store_A
 send s y ; % s : store_A
 ...)

- Even in a languages like Go, channels have a fixed type
- But see Ferrite session type library for Rust!
Sample Rules (External Choice)

\[
\Delta \vdash P_\ell :: (x : A_\ell) \quad (\forall \ell \in L)
\]

\[
\Delta \vdash \text{recv} \times (\ell \Rightarrow P_\ell)_{\ell \in L} :: (x : \&\{\ell : A_\ell\}_{\ell \in L})
\]

\[
k \in L \quad \Delta, x : A_k \vdash Q :: (z : C)
\]

\[
\Delta, x : \&\{\ell : A_\ell\}_{\ell \in L} \vdash \text{send} \times k ; Q :: (z : C)
\]
Preservation and Progress

- A configuration is a collection of semantic objects \( \text{proc}(P) \).
- Dynamics specified using multiset rewriting:

\[
\text{proc}(\text{recv } c (\ell \Rightarrow P_\ell)_{\ell \in L}), \text{proc}(\text{send } c k ; Q) \quad (k \in L) \quad \rightarrow \quad \text{proc}(P_k), \quad \text{proc}(Q)
\]

- Type evolves from \( c : \&\{\ell : A_\ell\} \) to \( c : A_k \).
- Server and client agree on type change.
- \( c \) is a private channel between the two processes.
  - Action is internal to the configuration.
- Preservation (\( \equiv \) session fidelity) holds.
- Progress (\( \equiv \) deadlock freedom) also holds.
Did we back ourselves into a corner?

- A lot of communication is **not synchronous**
- A lot of computation is **not linear** (eg, reuses data)
- A lot of communication is **not dyadic** (eg, multicast)
- Fortunately, the principles of (local) session types extend
- Generalize from synchronous/linear/dyadic
Step 1: Asynchronous Communication

- Messages as processes
- Requires *continuation channels* for type safety
- Example: internal choice
  - From

\[
\Delta, x : A_\ell \vdash Q_\ell :: (z : C) \quad (\forall \ell \in L)
\]

\[
\Delta, x : \bigoplus_{\ell \in L} A_\ell \vdash \text{recv} \times (\ell \Rightarrow Q_\ell)_{\ell \in L} :: (z : C)
\]

- To

\[
\Delta, x' : A_\ell \vdash Q_\ell(x') :: (z : C) \quad (\forall \ell \in L)
\]

\[
\Delta, x : \bigoplus_{\ell \in L} A_\ell \vdash \text{recv} \times (\ell(x') \Rightarrow Q_\ell(x')) :: (z : C)
\]

- Right rule now types a message as process

\[
k \in L
\]

\[
x' : A_k \vdash \text{send} \times k(x') :: (x : \bigoplus_{\ell \in L} A_\ell)
\]
Step 1: Asynchronous Dynamics

- Message has continuation channel
- Receiver has a continuation process

\[
\text{proc(}\text{send } c \ k(c'))), \ \text{proc(}\text{recv } c \ (\ell(x') \Rightarrow Q_\ell(x'))_{\ell \in L}) \ (k \in L) \\
\rightarrow \\
\text{proc(}Q_k(c')\text{)}
\]

- We can still track the provenance of a channel
- Ultimately yields data layout, functionally
Example Revisited

\[ \text{store}_A = \&\{ \text{ins} : A \to \text{store}_A, \]

\[ \text{del} : \oplus\{ \text{none} : 1, \text{some} : A \otimes \text{store}_A \} \} \]

\[ \text{server} :: (s : \text{store}_A) = \]

\[ \text{recv} \ s \ (\text{ins}(s')) \Rightarrow \% \ s' : A \to \text{store}_A \]

\[ \text{recv} \ s' \ ((x, s'') \Rightarrow \% \ s'' : \text{store}_A \]

\[ \ldots) \]

\[ | \text{del}(s') \Rightarrow \% \ s' : \oplus\{ \text{none} : 1, \text{some} : A \otimes \text{store}_A \} \]

\[ \text{send} \ s' \ \text{some}(s'') ; \% \ s'' : A \otimes \text{store}_A \]

\[ \text{send} \ s'' \ (y, s'''') ; \% \ s'' : \text{store}_A \]

\[ \ldots \) \]
Step 2: Multicast

- Distinguish linear channels $x_L$ and nonlinear channels $x_S$
- Distinguish ephemeral semantic objects $\text{proc}(P)$, $\text{msg}(P)$ and persistent semantic objects $!\text{msg}(P)$.
  - Ephemeral objects are consumed during transitions
  - Persistent objects are subject to garbage collection
- We can model multicast using persistent messages
- Sample rules: internal choice / sending a label

\[
\begin{align*}
\text{proc}(\text{send } c_L k(c_L')) & \rightarrow \text{msg}(\text{send } c_L k(c_L')) \\
\text{msg}(\text{send } c_L k(c_L')) & , \text{proc}(\text{recv } c_L (\ell(x_L') \Rightarrow Q_{\ell}(x_L'))_{\ell}) \rightarrow \text{proc}(Q_k(c_L'))
\end{align*}
\]

\[
\begin{align*}
\text{proc}(\text{send } c_S k(c_S')) & \rightarrow !\text{msg}(\text{send } c_S k(c_S')) \\
!\text{msg}(\text{send } c_S k(c_S')) & , \text{proc}(\text{recv } c_S (\ell(x_S') \Rightarrow Q_{\ell}(x_S'))_{\ell}) \rightarrow \text{proc}(Q_k(c_S'))
\end{align*}
\]
Step 2: Shared Service

- Symmetric with multicast
- The server is now persistent, not the message
- Spawns a fresh copy of itself upon message receipt
- Sample rules: external choice / receiving a label

\[
\text{proc}(\text{recv} \ c_s (\ell(x'_s) \Rightarrow P_{\ell}(x'_s))) \longrightarrow !\text{srv}(\text{recv} \ c_s (\ell(x'_s) \Rightarrow P_{\ell}(x'_s)))
\]

\[
!\text{srv}(\text{recv} \ c_s (\ell(x'_s) \Rightarrow P_{\ell}(x'_s))), \text{msg}(\text{send} \ c_s \ k(c'_s)) \longrightarrow \text{proc}(P_k(c'_s))
\]

- We can still track provenance
Step 3: Combining Linear and Nonlinear Types

- We use **shift** to mediate between linear and nonlinear layers

  Nonlinear  \( A_S ::= A_S \to B_S \mid A_S \times B_S \mid \ldots \mid \uparrow A_L \)

  Linear  \( A_L ::= A_L \leftarrow B_L \mid A_L \otimes B_L \mid \ldots \mid \downarrow A_S \)

- No need to distinguish the syntax of types or processes
- The mode signifies dyadic or variadic channel
- Mode determines:
  - Garbage collection for nonlinear processes and messages
  - No garbage collection for linear processes and messages
- This difference is **significant**
Starting point:
- Synchronous linear session types
- Channel type evolves during communication

Now:
- Asynchronous session types with continuation channels
- Combined linear (no gc) and nonlinear (with gc)
- Types do not evolve, due to continuation channels
- Provenance can be tracked

Next:
- What’s the connection to ordinary types?
Process Composition

- Process composition $x_m \leftarrow P(x) ; Q(x)$
- Dynamics (for linear $x$ and $a$)

$$\text{proc}(x \leftarrow P(x) ; Q(x)) \rightarrow \text{proc}(P(a)), \text{proc}(Q(a)) \quad \text{a fresh}$$

- Statics (all variables and propositions linear except $\Gamma_S$)

$$\Gamma_S, \Delta \vdash A \quad \Gamma_S, \Delta', A \vdash C$$

$$\frac{}{\Gamma_S, \Delta, \Delta' \vdash C} \text{cut}$$

$$\Gamma_S, \Delta \vdash P(x) :: (x : A) \quad \Gamma_S, \Delta', x : A \vdash Q(x) :: (z : C)$$

$$\frac{}{\Gamma_S, \Delta, \Delta' \vdash (x \leftarrow P(x) ; Q(x)) :: (z : C)} \text{cut}$$
Compiling Functional Programs

- At this point, session types \(\sim\) ordinary types
- Compile functional expressions with a destination \(d\)

\[ \llbracket e \rrbracket d = P \]

where \(\Gamma \vdash e : A_m\) implies \(\Gamma \vdash \llbracket e \rrbracket d :: (d : A_m)\)
- Translation is compositional

\[
\llbracket e_1 \ e_2 \rrbracket d =
\begin{align*}
x_1 & \leftarrow \llbracket e_1 \rrbracket x_1 ; \\
x_2 & \leftarrow \llbracket e_2 \rrbracket x_2 ; \\
\text{send} & \ x_1 \ (x_2, d)
\end{align*}
\]

\[
\llbracket \lambda x. \ e \rrbracket d = \text{recv} \ d \ ((x, d') \Rightarrow \llbracket e \rrbracket d')
\]

\[
\llbracket x \rrbracket d = \text{fwd} \ d \ x
\]
- Example

\[
\llbracket \lambda x. \ x \rrbracket d = \text{recv} \ d \ ((x, d') \Rightarrow \text{fwd} \ d' \ x)
\]
Parallelism/concurrency is possible, but not necessary

Example: call-by-need

\[[e_1 \; e_2] \; d = x_1 \leftarrow [e_1] \; x_1 \; ; \quad \% \text{ run } [e_1] \; x_1 \text{ until it blocks on } x_1
x_2 \leftarrow [e_2] \; x_2 \; ; \quad \% \text{ suspend } [e_2] \; x_2
\textbf{send} \; x_1 \; (x_2, \; d) \quad \% \text{ pass } x_2 \text{ and } d \text{ to function } x_1\]

\[[\lambda x. \; e] \; d = \textbf{recv} \; d \; ((x, \; d') \Rightarrow [e] \; d')\]

\[[x] \; d \; = \textbf{fwd} \; d \; x\]

Can also represent call-by-value and futures
Circling back: so what is special?

1. Integration of global and local types
   - Global types \( \sim \) specifications
   - Local types \( \sim \) implementations

2. Substructural (linear or affine) types
   - Reflect process state
   - Channel types evolve during communication

3. Revise and extend
   - Asynchronous communication
   - Continuation channels (with channel provenance)
   - Nonlinear types (shared servers and multicast)
   - Combining linear and nonlinear types

4. Import to “ordinary” functional programming
   - With futures, call-by-need, call-by-value
   - Cannibalized session types for mixed linear/nonlinear types (significant for memory (re)use)
   - Cannibalized continuation channels for data layout
What I have learned

- The significance of linear types
- The significance of mixed linear/nonlinear types
- The elegance of futures
- The connection between channel provenance and data layout
What I still don’t know

- Fundamentally, what are global session types?
- How are they connected to local session types?
- What does this mean beyond process communication?