Concurrent Programming in Linear Type Theory

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Outline

- A new foundation for session types
- SILL by example
  - Prime sieve
  - Bit strings
- Language highlights
  - Types and programs
  - Implementation
  - Ongoing research

Session Types

- Prescribe communication behavior between message-passing concurrent processes
- May be synchronous or asynchronous
- Linear channels with two endpoints
- Shared channels with multiple endpoints
- Messages exchanged can be
  - data values (including process expressions)
  - channels (as in the π-calculus)
  - labels (to indicate choice)

Curry-Howard Isomorphisms

- Logical origins of computational phenomena
- Intuitionistic logic ⇔ functional programming
- S4 modal logic ⇔ quote/eval staging
- S5 modal logic ⇔ distributed programming
- Temporal logic ⇔ partial evaluation
- Linear logic ⇔ session-typed concurrency
- More than an analogy!

Linear Logic: A New Foundation

- Linear propositions ⇔ session types
- Sequent proofs ⇔ process expressions
- Cut ⇔ process composition
- Identity ⇔ message forwarding
- Proof reduction ⇔ communication
- Linear type theory generalizes linear logic
  - Logic: propositions do not mention proofs
  - Type theory: proofs are internalized as terms

Benefits of Curry-Howard Design

- Integrated development of programming constructs and reasoning principles
  - Correct programs via simple reasoning principles
  - Even if they are not formalized in the language!
- Elegant and expressive language primitives
- Orthogonality and compatibility of constructs
- Programming language theory as proof theory

Curry-Howard: How Far to Go?

- Computation vs. proof reduction
  - Computation imposes a strategy
  - Proof reduction could be anywhere
  - η-expansion as equality, not computation
- Functional programming
  - Always stop at λ-abstraction (negative type)
  - Call-by-name vs. call-by-value vs. call-by-need vs…

Curry-Howard: How Far to Go?

- Option 1: Synchronous π-calculus
  - Only judgmental rules (cut, id) commute
  - No propositional rules commute
- Option 2: Asynchronous π-calculus
  - Commute past outputs (pos. multiplicatives)
  - Don't commute past inputs (as in functional progs)
- Option 3: Solos [N. Guénot yesterday]
  - Commute past inputs (neg. multiplicatives)
  - Do not commute past neg. additives, exponentials

Some Choices for SILL

- SILL = Sessions in Intuitionistic Linear Logic
- Conservatively extend functional language
  - Process expressions form a (contextual) monad
  - Communication may be observable
- Manifest notion of process
  - Offer vs. use of a service
  - Process ⇔ channel along which service is offered
- Later: CILL, sessions in a C-like language

Properties of SILL

- Type preservation
  - Entails session fidelity on processes
- Progress
  - Absence of deadlock
  - Absence of race conditions
- Termination and productivity
  - Some restrictions on recursive types required
- Obeys a general theory of logical relations!

SILL by Example

- Syntax close to implementation in O'Caml
- No inference rules, just intuition
- Examples
  - Endless streams of integers
  - Streams of integers
  - Stream filter
  - Prime sieve
  - Bit strings
  - Increment and addition

Stream of Numbers

- Data types

  $\tau ::= \text{bool} \mid \text{int} \mid \tau_1 \rightarrow \tau_2 \mid \ldots \mid \{ A \}$

- { A } is type of process offering service A
- Session types

  $A ::= \ldots$

- Data and session types may be recursive
- In type theory, should be inductive or coinductive (ongoing work)

Endless Streams of Integers

```
ints = int ∧ ints;

from : int → {ints};
c ← from n =
  send c n ;
  c ← from (n+1)
```

- $c : \tau \wedge A$ send value $v{:}\tau$ along c and behave as A
- Non-dependent version of $\exists x{:}\tau.\ A$
- Tail call represents process continuation
- A single process will send stream of integers
- Channel variables and session types in red

Streams of Integers

```
ints = &{next:int ∧ ints, stop:1};

from : int → {ints};
c ← from n =
  case (recv c)
   | next ⇒ send c n ;
            c ← from (n+1)
   | stop ⇒ close c
```

- $c : \&\{l_i : A_i\}_i$ receive label l_i along c and continue as A_i
- Labeled n-ary version of linear logic A & B
- External (client's) choice
- $c : 1$ terminate process; as linear logic 1
- Closing a channel c terminates offering process

Filtering a Stream

```
ints = &{next:int ∧ ints, stop:1};
filter : (int → bool) → {ints ← ints};
filterNext : (int → bool) → {int ∧ ints ← ints};

c ← filter q ← d =
  case (recv c)
   | next ⇒ c ← filterNext q ← d
   | stop ⇒ send d stop ;
             wait d ;
             close c
```

- {A ← A_1, …, A_n} process offering A, using A_i’s
- Type of channels changes based on process state!
- Type error, say, if we forget to stop d

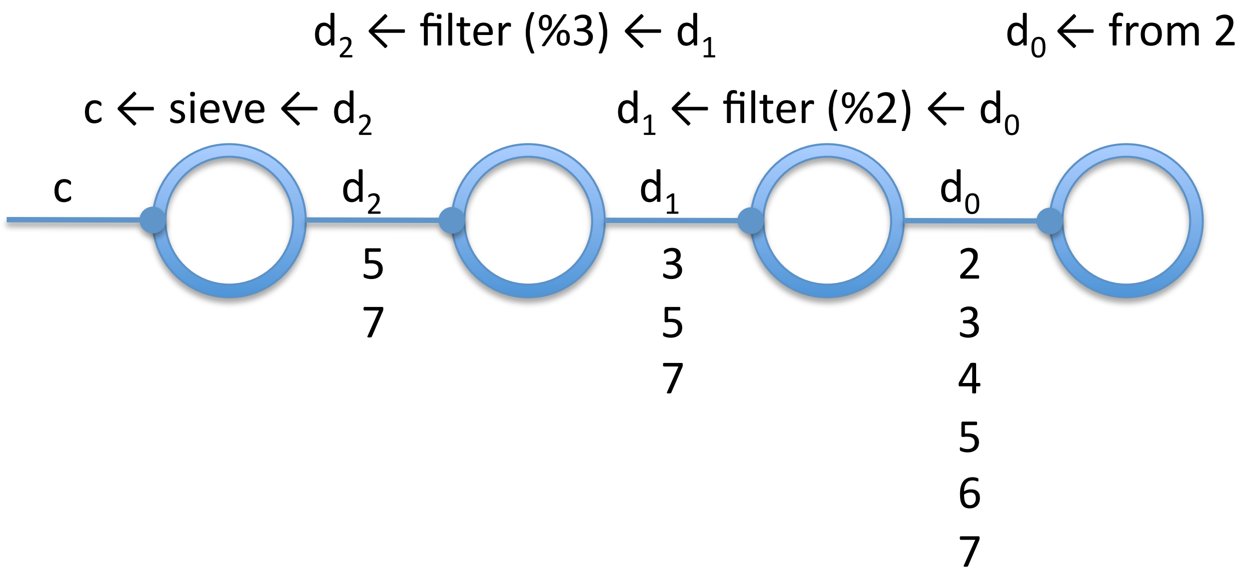
Finding the Next Element

```
ints = &{next:int ∧ ints, stop:1};
filter : (int → bool) → {ints ← ints};
filterNext : (int → bool) → {int ∧ ints ← ints};

c ← filterNext q ← d =
  send d next ;
  n ← recv d ;
  case (q n)
   | true ⇒ send c n ;
            c ← filter q ← d
   | false ⇒ c ← filterNext q ← d
```

- filter/filterNext process identified with channel c

Prime Sieve



d_2 ← filter (%3) ← d_1

d_0 ← from 2

c ← sieve ← d_2

d_1 ← filter (%2) ← d_0

- c ← sieve ← d sends first value p on d along c
- Then spawns new process to filter out %p

Prime Sieve

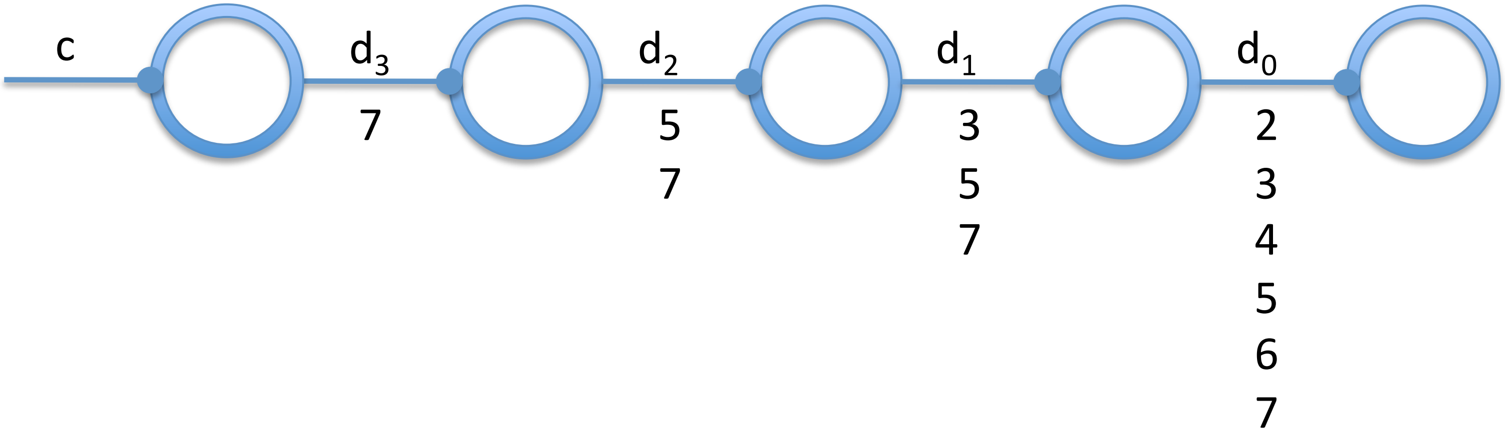
c ← sieve ← d_3

d_2 ← filter (%3) ← d_1

d_0 ← from 2

d_3 ← filter (%5) ← d_2

d_1 ← filter (%2) ← d_0



- c ← sieve ← d sends first value p on d along c
- Then spawns new process to filter out %p

Prime Sieve

```
ints = &{next:int ∧ ints, stop:1};
sieve : {ints ← ints};

c ← sieve ← d =
 case (recv c)
 | next ⇒ send d next ;
          p ← recv d ;
          send c p ;
          e ← filter (mod p) ← d ;
          c ← sieve ← e
 | stop ⇒ send d stop ; wait d ; close c
```

- e ← filter (mod p) ← d spawns new process
- Uses d, offers e (which is used by sieve)

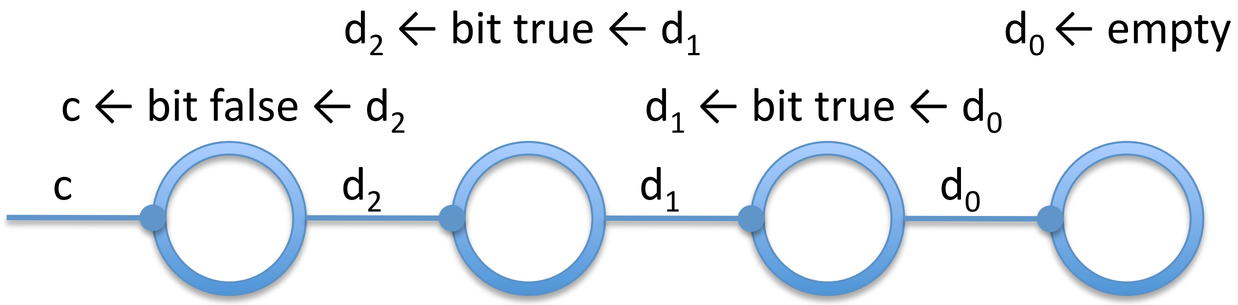
Primes

```
ints = &{next:int ∧ ints, stop:1};
primes : {ints};

c ← primes =
  d ← from 2 ;
  c ← sieve ← d
```

- Primes correct with sync or async communication
- n+2 processes for n primes

Bit Strings



$d_2 \leftarrow$ bit true $\leftarrow d_1$

$d_0 \leftarrow$ empty

$c \leftarrow$ bit false $\leftarrow d_2$

$d_1 \leftarrow$ bit true $\leftarrow d_0$

```
bits = ⊕{eps:1, bit:bool ∧ bits};
```

- Lowest bit on the left (above represents 6)
- $c : \oplus\{l_i{:}A_i\}_i$ send a label l_i along c and cont. as A_i
- n-ary version of linear logic $A \oplus B$
- Internal (provider's) choice

Bit String Constructors

```
bits = ⊕{eps:1, bit:bool ∧ bits};

empty : {bits};
c ← empty =
  send c eps ;
  close c

bit : bool → {bits ← bits};
c ← bit b ← d =
  send c bit ;
  send c b ;
  c ← d;
```

- Forwarding c ← d represents logical identity
  - Process offering along c terminates
  - Client subsequently talks to process offering along d

Alternative Constructor

```
bits = ⊕{eps:1, bit:bool ∧ bits};

num : int → {bits};
c ← num n =
case n == 0
| true ⇒ send c eps ; close c
| false ⇒ send c bit ;
           send c (odd n) ;
           c ← num (n/2)
```

- num as a single process holding an int n
- Channel type is process interface, not representation

Increment

```
bits = ⊕{eps:1, bit:bool ∧ bits};
inc : {bits ← bits};

c ← inc ← d =
  case (recv d)
  | eps ⇒ wait d ;
          e ← eps ;
          c ← bit true ← e
  | bit ⇒ b ← recv d ;
          case b
           | true ⇒ e ← inc ← d ;
                    c ← bit false ← e
           | false ⇒ c ← bit true ← d
```

- inc process generates one bit string from another
- Spawns a new inc process in case of a carry

Addition

```
bits = ⊕{eps:1, bit:bool ∧ bits};
add : {bits ← bits, bits};
c ← add ← d, e =
  case (recv d)
  | eps ⇒ wait d ;
          c ← e
  | bit ⇒ b1 ← recv d ;
          case (recv e)
          | eps ⇒ wait e ;
                  send c bit;
                  send c b1;
                  c ← d
          | bit ⇒ b2 ← recv e ; …
```

- add uses two channels, provides one
- Receives are sequential; additional parallelism could be justified by commuting conversions in proof theory

Other Examples

- Data structures
  - Stacks, queues, binary search trees
  - Syntax trees, evaluation, tree transformation
- Algorithms
  - Lazy and eager prime sieve
  - Merge sort, odd/even sort, insertion sort
- Protocols
  - Needham/Schroeder, safe and unsafe

Odd/Even Sort

```
cell  = ⊕{someR:int ∧ cell’, tail:cell};
cell’ = &{someL:int → cell, head:cell};
elem : side → int → int → {cell ← cell};
c ← elem _ 0      n ← d = … (sorted)
c ← elem L (i+1) m ← d =
  case (recv d)
  | someR ⇒ k ← recv d ;
              send d someL ; send d m ;
              case m > k
              | true  ⇒ c ← elem R i k ← d
              | false ⇒ c ← elem R i m ← d
  | tail ⇒ c ← elem R i m ← d
c ← elem R (i+1) k ← d =
  send c someR ; send c k ;
  case (recv c)
  | someL ⇒ m ← recv c ;
              case m > k
              | true  ⇒ c ← elem L i m ← d
              | false ⇒ c ← elem L i k ← d
  | head ⇒ c ← elem L i k ← d
```

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Session Type Summary

- From the point of view of session provider

$c : \tau \wedge A$	send value $v : \tau$ along c, continue as A
$c : \tau \rightarrow A$	receive value $v : \tau$ along c, continue as A
$c : A \otimes B$	send channel $d : A$ along c, continue as B
$c : A \multimap B$	receive channel $d : A$ along c, continue as B
$c : 1$	close channel c and terminate
$c : \oplus\{l_i : A_i\}$	send label l_i along c, continue as A_i
$c : \&\{l_i : A_i\}$	receive label l_i along c, continue as A_i
$c : !A$	send persistent $!u : A$ along c and terminate
$!u : A$	receive $c : A$ along $!u$ for fresh instance of A

Contextual Monad

- $M : \{ A \leftarrow A_1, \ldots, A_n \}$ process expressions offering service A, using services $A_1, \ldots, A_n$
- Composition $c \leftarrow M \leftarrow d_1, \ldots, d_n ; P$
  - c fresh, used (linearly) in P, consuming $d_1, \ldots, d_n$
- Identity $c \leftarrow d$
  - Notify client of c to talk to d instead and terminate
- Strong notion of process identity

Static Type Checking

- Bidirectional
  - Precise location of type errors
  - Based on definition of normal proofs in logic
  - Fully compatible with linearity
- Natural notion of behavioral subtyping, e.g.
  - $\&\{l{:}A, k{:}B\} \leq \&\{l{:}A\}$ (we can offer unused alt's)
  - $\oplus\{l{:}A\} \leq \oplus\{l{:}A, k{:}B\}$ (we need not produce all alt's)
- Supports ML-style value polymorphism
- No behavioral polymorphism yet

Dynamic Semantics

- Three back ends
  - Synchronous threads
  - Asynchronous threads
  - Distributed processes
- Fourth back end (hypothetical):
  - Solos ?
- Curry-Howard lesson:
  - The syntax can remain stable (proofs!)
  - The semantics can vary: controling reductions
  - Must be consistent with proof theory
- Not released (but multiple “friendly” users)

Dynamic Type Checking

- May not trust all participating processes
- Type system compatible with
  - Value dependent types, e.g. nat = $\{x{:}int \mid x \geq 0\}$
  - Full dependent types, but still under investigation:
    - "Right" equivalence on process expressions
    - Restrictions on recursive types
- Contracts are partial identity processes
  - Blame assignment (ongoing)
  - Causality (ongoing)

Some Refinements

```
nat = {x:int | x ≥ 0};
nats = &{next:nat ∧ nats, stop:1};

eq n = {x:int | x = n};
succs n = &{next:eq n ∧ succs(n+1), stop:1};

gt n = {x:int | x > n};
incrs n = &{next:∃k:gt n. incrs k, stop:1};
```

- eq and gt are value type families
- succs and incrs are session type families
- Last line illustrates ∃ as dependent ∧
- Not yet implemented

Other Logical Thoughts

- Affine logic (= linear logic + weakening)
  - Static deallocations inserted
  - Shorter programs, but errors more likely
- Hybrid linear logic (= linear logic + worlds)
  - Worlds representing security domains
  - Accessibility relation between domains
  - Ongoing
- Affirmation modality for digital signatures

Session Types in a C-like Language

- C0: a type-safe subset of C
  - Designed for teaching imperative programming, algorithms, and data structures to freshmen
  - Extended with contracts (pure boolean functions)
  - Contracts are crucial for design, proof, and testing
- C1: function pointers and polymorphism
- CILL: session-typed concurrency?

CILL

- Channels \$c are linearly typed (as in SILL)
- Persistent channels \$\$c, variables x as usual
- Channel types must be loop invariants
  - lub at all join points in control-flow graph
- Possible with or without shared memory
  - No safety in the presence of shared memory
- Exploring robustness of SILL concepts in different setting

Integer Streams in CILL

```
choice intstream {
  int /\ choice intstream next;
  void stop;
};
typedef choice intstream ints;

ints $c from(int n) {
  while (true) {
    switch ($c) {
    case next:
      send($c, n);
      n = n+1;
    case stop:
      close($c);
    }
  }
}
```

Speculating on Contracts

```
ints $c from(int n)
//@requires n >= 0;
//@ensures $c = all_pos($c);
{
  while (true) {
    switch ($c) {
    case next:
      send($c, n);
      n = n+1;
    case stop:
      close($c);
    }
  }
}
```

- Value contracts must be pure boolean functions
- Channel contracts must be partial identity proc’s

Partial Identity Process

```
ints $c all_pos(ints $d) {
  switch ($c) {
  case next:
    $d.next;
    int n = recv($d);
    if (n <= 0) abort;
    send($c, n);
    $c = all_pos($d);
  case stop:
    $d.stop; wait($d);
    close($c);
  }
}
```

- Synthesized in a type-directed way

Summary

- SILL, a functional language with a contextual monad for session-typed message-passing concurrency
  - Type preservation (session fidelity)
  - Progress (deadlock and race freedom)
  - Implementation with subtyping, polymorphism, recursive types
- Based on a Curry-Howard interpretation of intuitionistic linear logic
- Full dependent type theory in progress

Some References

- 2010
  - CONCUR: the basic idea, revised for MSCS, 2012
- 2011
  - PPDP: dependent types
  - CPP: digital signatures (♢A)
- 2012
  - CSL: asynchronous comm.
  - ESOP: logical relations
  - FOSSACS: functions as processes
- 2013
  - ESOP: behavioral polymorphism
  - ESOP: monadic integration (SILL)
- 2014 (in progress)
  - Security domains (A @ w), spatial distribution
    - J. Peréz, 14:30 today!
  - Coinductive types
  - Blame assignment

Thanks!

- Luís Caires, Bernardo Toninho, Jorge Peréz (Universidade Nova de Lisboa)
  - FCT and CMU|Portugal collaboration
- Dennis Griffith, Elsa Gunter (UIUC) [Implementation]
  - NSA
- Michael Arntzenius, Limin Jia (CMU) [Blame]
- Stephanie Balzer (CMU) [New foundation for OO]
- Henry DeYoung (CMU) [From global specs to local types]
- Much more to say; see http://www.cs.cmu.edu/~fp
- Apologies for the lack of references to related work