Possession as Linear Knowledge

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Goals

- Logical specification of distributed authorization policies
- Reliable enforcement of such high-level policies
- Mechanized reasoning about consequences of policies:
  - Evolution of system state
  - Principals' knowledge (information)
  - Principals' possessions (consumable resources)

Approach: linear epistemic logic

Examples:
- Documents in the intelligence community of the US
- Course management
- Monetary instruments
- File system
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Understanding Distributed Systems

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1. **Background: Proof-Carrying Authorization**

2. **Logical Foundations**
   - 1. Resources (linear logic)
   - 2. Possessions (linear epistemic logic)
   - 3. Effects (linear lax logic)
   - 4. From axioms to inference rules via focusing
   - 5. Persistent truth and knowledge (epistemic logic)

3. **Policy Consequences**
   - 1. State invariants
   - 2. Proving metatheorems

4. **Speculation: linear epistemic logic programming**
Logic for distributed authorization

- Authorization policy is stated as a logical theory $T$
- Principal $K$ can perform operation $O$ if authorization proposition $\text{may}(K, O)$ is true in $T$
- The proof embodies the reason why action should be permitted

Core: "$K$ says $A$" for principal $K$ and proposition $A$

- Family of $K$-indexed modal operators
- Precise definition not important for this talk
Background: Proof-Carrying Authorization

- Enforcement architecture for access control
- “K says A” can be realized in two ways
  - Proposition “A” digitally signed by K
  - Explicit proof using logical inference
- Policy theory consists of signed “K says A”
- Reference monitor grants access if formal proof object “M : K says may(L, O)” is correct (for resource owner K)
- Core: Proof checking and certificate verification
- Examples
  - Gray (office access with smartphones)
  - Nexus (document viewer application suite)
  - PCFS (proof-carrying file system)
Example: A Versioned File System

Principals \( K, L \): \( fs, \ldots \)
Operations \( O \): create, on(\( F, A \))
Actions \( A \): read, write(s), delete
Propositions:
\[
\langle fs \rangle \text{user}(K) \\
\langle fs \rangle \text{owns}(K, F) \\
\langle fs \rangle \text{may}(L, O), \langle K \rangle \text{may}(L, O)
\]

Sample policy, file system

create : \( \langle fs \rangle (\text{user}(K) \supset \text{may}(K, \text{create})) \)
delegate : \( \langle fs \rangle (\text{owns}(K, F) \land \langle K \rangle \text{may}(L, \text{on}(F, A))) \supset \text{may}(L, \text{on}(F, A))) \)
Example: Distributed Policy

Sample policy, Alice

\[ \langle \text{alice} \rangle (\langle \text{fs} \rangle \text{owns}(\text{alice}, F)) \supset \text{may}(\text{alice}, \text{on}(F, A))) \]

\[ \langle \text{alice} \rangle (\text{friend}(K, \text{alice})) \supset \text{may}(K, \text{on}(\text{embarassing.jpg, read})) \]

\[ \langle \text{alice} \rangle (\text{friend}(K, \text{alice}) \land \langle K \rangle \text{friend}(L, K)) \supset \text{may}(L, \text{on}(\text{fun.jpg, read}))) \]
Access to or with consumable resources
- “K says pay(K, L, $50)”
- “netflix says may(L, playmovie(3))”

Core: linear authorization logic

Enforcement
- Linear digitally signed certificates
- Linear proof checking
- Reference counting in resource monitor

Atomicity: multi-party contract signing
Semantics

- Capture consequences of authorization policy
  - Information flow: what **knowledge** may principals gain?
  - Accounting: what **possessions** may principals obtain or relinquish?

- Which states of knowledge and possession can be reached?

- Verify desirable semantic consequences
  - “To learn the contents of a file, one must have read or write access”
  - “Banking machines fees for a single transaction will be no more than $2”
  - “Every valid electronic vote will be counted”

- Caveat: we stay within the level of abstraction of the semantic description
Example: File System State

Command:  ⟨K⟩do(K, O)  linear
Version:   [K]current(F, V)   possession – linear
Contents:  [K]contains(F, V, S)   knowledge – persistent

Sample rule: Creating a file

⟨K⟩do(K, create)
⊗ ⟨fs⟩may(K, create)
→ {∃f. ∃v.
!⟨fs⟩owns(K, f)
⊗ [fs]current(f, v)
⊗ [fs]contains(f, v, "")
⊗ [K]contains(f, v, "")}
Example: Reading a File

\( \langle K \rangle \text{do}(K, \text{on}(F, \text{read})) \)
\( \otimes \langle fs \rangle \text{may}(K, \text{on}(F, \text{read})) \)
\( \otimes [fs] \text{current}(F, V) \)
\( \otimes [[[fs]] \text{contents}(F, V, S)] \)
\( \rightarrow \{[[fs] \text{current}(F, V)] \)
\( \otimes [K] \text{contents}(F, V, S) \} \)

Key to Syntax

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle K \rangle A )</td>
<td>“K says A”</td>
</tr>
<tr>
<td>[K]A</td>
<td>“K has A”</td>
</tr>
<tr>
<td>[[K]]A</td>
<td>“K knows A”</td>
</tr>
<tr>
<td>{A}</td>
<td>“A, with effect”</td>
</tr>
</tbody>
</table>
Example: Writing to a File

\[
\langle K \rangle \text{do}(K, \text{on}(F, \text{write}(S)))
\]

\[
\otimes \langle fs \rangle \text{may}(K, \text{on}(F, \text{write}(S)))
\]

\[
\otimes [fs] \text{current}(F, V)
\]

\[\implies \exists v'. [fs] \text{current}(F, v')\]

\[
\otimes [fs] \text{contains}(F, v', S)
\]

\[
\otimes [K] \text{contains}(F, v', S)\}
\]

Key to Syntax

\[
\langle K \rangle A = "K says A"
\]

\[
[K]A = "K has A"
\]

\[
[[K]]A = "K knows A"
\]

\[
\{A\} = "A, with effect"
\]
Example: Deleting a File

\[
\langle K \rangle \text{do}(K, \text{on}(F, \text{delete}))
\]
\[
\otimes \langle fs \rangle \text{may}(K, \text{on}(F, \text{delete}))
\]
\[
\otimes [fs] \text{current}(F, V)
\]
\[
\rightarrow \{1\}
\]
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4. Speculation: linear epistemic logic programming
Goal: define a suitable linear logic of (authorization), possession, knowledge, and effects — linear epistemic logic

Use such a logic

- Logically: specifying the consequences of authorization policies
- Metalogically: reasoning about all possible action sequences
- Operationally: implementing (or checking implementation against) linear epistemic specification
Proof-Theoretic Semantics

- How do we define the right logic?
- The crucial role of **proofs**
  - Explicit evidence for authorization
  - Explicit evidence for right-to-know
  - Explicit evidence for transactions
  - Explicit traces of system evolution
- In combination with cryptographic techniques
  - Digital signatures
  - Encryption and decryption
Judgments and Propositions

linear sequent

$A_1 \text{ res}, \ldots, A_n \text{ res}$ $\implies$ $C \text{ true}$

$\Delta$

consumable resources

linear assumptions

antecedents

$\gamma$

goal

conclusion

succedent
Judgmental Principles

Identity: With resource \( A \) we can achieve goal \( A \)

\[
\begin{align*}
\text{id}_A & \quad A \text{ res} \implies A \text{ true} \\
\end{align*}
\]

Cut: If we can achieve \( A \) we can use it as a resource

\[
\begin{align*}
\Delta & \implies A \text{ true} \quad \Delta', A \text{ res} \implies \gamma \\
\text{cut}_A & \quad \Delta, \Delta' \implies \gamma \\
\end{align*}
\]

- These must be admissible rules (metatheorems)
- Harmony between resources and goals
Simultaneous Conjunction $A \otimes B$

- Right rule: how to prove goal can be achieved

\[
\begin{align*}
\Delta_A \rightarrow A & \quad \Delta_B \rightarrow B \\
\hline
\Delta_A, \Delta_B \rightarrow A \otimes B & \quad \otimes R
\end{align*}
\]

- Left rule: how to use resource

\[
\begin{align*}
\Delta, A, B \rightarrow \gamma \\
\hline
\Delta, A \otimes B \rightarrow \gamma & \quad \otimes L
\end{align*}
\]

(Elide \textit{res} and \textit{true} since clear from position)
Local Harmony

- Show how to expand

\[ A \implies A \implies_{E} \text{id}_{A} \]

using identity on subformulas of \( A \)

- Part of proof of global identity proof by induction on \( A \)
- Need primitive rule \( P \implies P \) for atomic \( P \)

- Show how to reduce

\[
\begin{align*}
\Delta & \implies A \\
\Delta' & \implies A \\
\Delta, \Delta' & \implies \gamma
\end{align*}
\]

using cut on subformulas of \( A \)

- Part of global cut proof by nested induction on \( A, \mathcal{D}, \mathcal{E} \)
Identity expansion

\[ A \otimes B \Rightarrow A \otimes B \]

\[ \text{id}_{A \otimes B} \]

\[ A \Rightarrow A \]

\[ B \Rightarrow B \]

\[ A, B \Rightarrow A \otimes B \]

\[ \text{id}_A \]

\[ \text{id}_B \]

\[ \otimes R \]

\[ \otimes L \]
Local Harmony for $A \otimes B$

- Cut reduction

\[
\begin{align*}
&D_A \quad D_B \\
&\Delta_A \Rightarrow A \quad \Delta_B \Rightarrow B \\
&\Delta_A, \Delta_B \Rightarrow A \otimes B \\
&\otimes R \\
&\Delta, \Delta_A, \Delta_B \Rightarrow \gamma
\end{align*}
\]

\[
\begin{align*}
&E \\
&\Delta, A, B \Rightarrow \gamma \\
&\otimes L \\
&\Delta, A \otimes B \Rightarrow \gamma
\end{align*}
\]

\[
\begin{align*}
&\text{cut}_{A \otimes B}
\end{align*}
\]

\[
\begin{align*}
&D_A \\
&\Delta_A \Rightarrow A \\
&\Delta, \Delta_A, B \Rightarrow \gamma \\
&\text{cut}_A
\end{align*}
\]

\[
\begin{align*}
&D_B \\
&\Delta_B \Rightarrow B \\
&\Delta, \Delta_A, \Delta_B \Rightarrow \gamma \\
&\text{cut}_B
\end{align*}
\]
Linear Implication $A \multimap B$

- **Right rule:** how to prove $A \multimap B$
  \[
  \Delta, A \implies B \\
  \Delta \implies A \multimap B \\
  \rightarrow \multimap R
  \]

- **Left rule:** how to use $A \multimap B$
  \[
  \Delta_A \implies A \quad \Delta_B, B \implies \gamma \\
  \Delta_A, \Delta_B, A \multimap B \implies \gamma \\
  \rightarrow \multimap L
  \]
Identity Expansion for $A \rightsquigarrow B$

$A \rightsquigarrow B \Rightarrow A \rightsquigarrow B \xrightarrow{id_{A \rightarrow B}} E \xrightarrow{id_A} A \Rightarrow A$

$B \Rightarrow B \xrightarrow{id_B} B \Rightarrow L$

$A \rightsquigarrow B, A \Rightarrow B \xrightarrow{id_A} A \rightsquigarrow B \Rightarrow A \rightsquigarrow B \xrightarrow{id_B} A \Rightarrow B \Rightarrow R$
Cut Reduction for $A \rightarrow B$

\[
\frac{\Delta, A \rightarrow B}{\Delta \Rightarrow A \rightarrow B} \quad \rightarrow R
\]

\[
\frac{\Delta_A \Rightarrow A}{\Delta, \Delta_A \Rightarrow B} \quad \frac{\Delta_B, B \Rightarrow \gamma}{\Delta_B, \Delta_B, A \rightarrow B \Rightarrow \gamma} \quad \rightarrow L
\]

\[
\Delta, \Delta_A, \Delta_B \Rightarrow \gamma
\]

\[
\frac{\Delta_A \Rightarrow A}{\Delta, \Delta_A \Rightarrow B} \quad \frac{\Delta, A \rightarrow B}{\Delta, \Delta_A, \Delta_B \Rightarrow \gamma} \quad \rightarrow R
\]

\[
\frac{\Delta_B, B \Rightarrow \gamma}{\Delta_B, \Delta_B, A \rightarrow B \Rightarrow \gamma} \quad \rightarrow L
\]

\[
\Delta, \Delta_A, \Delta_B \Rightarrow \gamma
\]
“●” denotes no resources
Example: Resources

- Example: $\$, $\$, $\$, ($\otimes \$ \rightsquigarrow \text{coffee}$) $\Rightarrow$ coffee $\otimes \$

\[
\begin{array}{c}
$ \Rightarrow $ \text{id} \\
$ \Rightarrow $ \text{id} \\
$ \Rightarrow $ \otimes R \\
$ \Rightarrow $ \text{id} \\
$ \Rightarrow $ \text{id} \\
$ \Rightarrow $ \otimes R \\
\end{array}
\]

- In a proof, all resources have to be used exactly once

\[
\begin{array}{c}
$ \Rightarrow $ \$ \otimes $ \\
$ \Rightarrow $ \$ \otimes $ \\
$ \Rightarrow $ \$ \otimes $ \\
\end{array}
\]

- $\$, $\$, $\$, ($\otimes \$ \rightsquigarrow \text{coffee}$) $\Rightarrow$ coffee $\otimes \$

- $\$, $\$, $\$, ($\otimes \$ \rightsquigarrow \text{coffee}$) $\Rightarrow$ coffee $\otimes \$

- $\$ $\otimes \$ \rightarrow \text{coffee}$ should be an axiom that we can use as often as we want
Example: Possession

- Previous example is imprecise: who has the dollars and who has the coffee? More precise ($tdo = Tazza D’Oro$)

$$[fp]\$ \otimes [fp]\$ \otimes [tdo]beans \rightarrow [fp]coffee \otimes [tdo]$\$ \otimes [tdo]$\$

- Need possession modality $[K]A$ ("K has A")
New judgment: $K$ has $A$ (used as assumption)

Judgmental rule: $K$ can relinquish possession

\[
\frac{\Delta, A \text{ res} \implies \gamma}{\Delta, K \text{ has } A \implies \gamma} \quad \text{has}_L
\]

$K$ cannot gain possession (arbitrarily)

Judgmental definition: (always silently expanded on right)

\[
\left[ \frac{\Delta|_K \implies A \text{ true}}{\Delta|_K \implies K \text{ has } A} \right] \quad \text{has}_R
\]

$\Delta|_K$ only has antecedents of the form “$K$ has $A$”
Identity and Cut

- No new identity principle

\[
\begin{align*}
\xi & : A \Rightarrow A \quad \text{id} \\
\frac{K \text{ has } A \Rightarrow A}{K \text{ has } A \Rightarrow K \text{ has } A} \quad \text{has}^R
\end{align*}
\]

- Derived cut principle

\[
\begin{align*}
\Delta|_K \Rightarrow A \\
\frac{\Delta|_K \Rightarrow K \text{ has } A}{\Delta', K \text{ has } A \Rightarrow \gamma} \quad \text{cut has}
\end{align*}
\]
Possession as a Proposition

- Internalize $K$ has $A$ judgment as a proposition $[K]A$

\[
\begin{align*}
\Delta |_K \Rightarrow A & \quad [ ] R \\
\Delta |_K \Rightarrow [K]A & \\
\Delta, K \text{ has } A \Rightarrow \gamma & \quad [ ] L
\end{align*}
\]
Identity Expansion for Possession

\[ [K]A \Rightarrow [K]A \quad \text{id}_{[K]A} \]

\[ [K]\text{has} A \Rightarrow A \quad \text{has}_L \]

\[ [K]\text{has} A \Rightarrow [K]A \quad \text{has}_R \]

\[ [K]A \Rightarrow [K]A \quad \rightarrow_E \]
Cut Reduction for Possession

\[
\begin{align*}
\mathcal{D} & \quad \frac{\Delta|_K \Longrightarrow A}{\Delta|_K \Longrightarrow [K]A} \\
[\ ] & \quad \frac{[\ ]R \quad \Delta', K \text{ has } A \Longrightarrow \gamma}{[\ ]L \quad \Delta', [K]A \Longrightarrow \gamma} \\
& \quad \frac{\Delta|_K, \Delta' \Longrightarrow \gamma}{\text{cut}_{[K]A}}
\end{align*}
\]

\[
\begin{align*}
\mathcal{D} & \quad \frac{\Delta|_K \Longrightarrow A}{\Delta|_K \Longrightarrow [K]A} \\
& \quad \frac{\Delta', K \text{ has } A \Longrightarrow \gamma}{\text{cut}_{K \text{ has } A}}
\end{align*}
\]
Axiomatics

- Axioms like Intuitionistic S4, but linear

\[ \vdash [K](A \rightarrow B) \rightarrow ([K]A \rightarrow [K]B) \quad (K\Box) \]
\[ \vdash [K]A \rightarrow [K][K]A \quad (4\Box) \]
\[ \vdash [K]A \rightarrow A \quad (T\Box) \]

- Rule of necessitation

\[ \vdash A \]

\[ \vdash [K]A \quad \text{(nec)} \]
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Applying rules such as

\[ [fp] \otimes [fp] \otimes [tdo] \text{beans} \rightarrow [fp] \text{coffee} \otimes [tdo] \otimes [tdo] \]

represent a change of state

Proofs of authorizations such as \langle fs \rangle \text{may}(K, \text{on}(F, \text{read})) do not involve a change of state

Isolate changes in an effect monad

Logically, this is a lax modality \{A\}

Rewrite above as

\[ [fp] \otimes [fp] \otimes [tdo] \text{beans} \rightarrow \{ [fp] \text{coffee} \otimes [tdo] \otimes [tdo] \} \]
Lax Judgment

- New judgment $A \textit{lax}$ ($A$ is true with effect)
- Judgmental rule: truth entails lax truth

$$\Delta \Longrightarrow A \textit{true} \quad \text{lax}_R$$
$$\Delta \Longrightarrow A \textit{lax}$$

- Lax truth does not entail truth
- Judgmental definition: (always silently expanded on the left)

$$\left[ \Delta, A \textit{res} \Longrightarrow C \textit{lax} \right] \quad \text{lax}_L$$

$$\Delta, A \textit{lax} \Longrightarrow C \textit{lax}$$

- Applies only with lax succedent, not truth
Judgmental Principles

- No new identity principle

\[
\begin{align*}
A \text{ res} & \implies A \text{ true} \\
A \text{ res} & \implies A \text{ lax}
\end{align*}
\]

\[\text{id}_A, \quad \text{lax}_R, \quad \text{lax}_L\]

- Derived cut principle

\[
\begin{align*}
\Delta', A \text{ res} & \implies C \text{ lax} \\
\Delta & \implies A \text{ lax} \\
\Delta', A \text{ lax} & \implies C \text{ lax}
\end{align*}
\]

\[\text{lax}_R, \quad \text{cut}_{\text{lax}}\]

- Allow \(\gamma ::= C \text{ true} \mid C \text{ lax}\) in all other rules with generic succedent
Lax Modality $\equiv$ Effect Monad

- Internalize lax judgment as proposition $\{A\}$

$$
\frac{\Delta \implies A \text{ lax}}{\Delta \implies \{A\} \text{ true}} \quad R
$$

$$
\frac{\Delta, A \text{ res} \implies C \text{ lax}}{\Delta, \{A\} \text{ res} \implies C \text{ lax}} \quad L
$$

- Identity expansion

$$
\frac{\text{id}_A}{A \implies A} \quad \text{lax}_L
$$

$$
\frac{\{A\} \implies A \text{ lax}}{\{A\} \implies A \text{ lax}} \quad L
$$

$$
\frac{\{A\} \implies \{A\}}{\{A\} \implies \{A\}} \quad E
$$
Cut Reduction for Lax Modality

\[ \begin{align*}
\mathcal{D} & \quad \Delta \Longrightarrow A \text{ lax} \\
\Delta & \Longrightarrow \{A\} \\
\mathcal{E} & \quad \Delta' \Longrightarrow A \Longrightarrow C \text{ lax} \\
\Delta', \{A\} & \Longrightarrow C \text{ lax} \\
& \quad \Delta, \Delta' \Longrightarrow C \text{ lax} \\
\mathcal{D} & \quad \Delta \Longrightarrow A \text{ lax} \\
\Delta' & \Longrightarrow A \Longrightarrow C \text{ lax} \\
\mathcal{E} & \quad \Delta, \Delta' \Longrightarrow C \text{ lax} \\
& \quad \text{cut}_{A \text{ lax}} \\
\end{align*} \]
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Polarization

- **Focusing**: we can obtain a complete big-step proof system using two observations
  - Apply invertible rules eagerly
  - When all top-level propositions have non-invertible rules, focus on one of them and apply a run of non-invertible rules to its components

- Robust technique (all reasonable known logics?)

- **Polarization**: we explicitly categorize propositions into negative (invertible right) and positive (invertible left).
  - Here: exploit monad (other choices are possible)

<table>
<thead>
<tr>
<th>Category</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative</td>
<td>$A^- ::= \ P^- \</td>
</tr>
<tr>
<td>Positive</td>
<td>$A^+ ::= A_1 \otimes A_2 \</td>
</tr>
</tbody>
</table>
Write $A$ for formula in focus
Must apply rule to focus formula

$\Delta, fp \textit{ has coffee}, tdo \textit{ has $} \Rightarrow C \textit{ lax} \hspace{1cm} [ ]L$

$\Delta, [fp]\textit{coffee}, [tdo]$ $\Rightarrow C \textit{ lax} \hspace{1cm} [ ]L$

$\Delta, [fp]\textit{coffee} \otimes [tdo]$ $\Rightarrow C \textit{ lax} \hspace{1cm} \otimes L$

$\Delta, \{[fp]\textit{coffee} \otimes [tdo]\} \Rightarrow C \textit{ lax} \hspace{1cm} \{ \}L$

\[
\begin{align*}
\frac{\text{id}}{\$ \Rightarrow \$} & \hspace{1cm} \text{has}_L \frac{\text{id}}{\text{beans} \Rightarrow \text{beans}} & \hspace{1cm} \text{has}_L \\
\frac{fp \textit{ has $} \Rightarrow \$}{fp \textit{ has $} \Rightarrow [fp]$} & \hspace{1cm} [ ]R \\
\frac{tdo \textit{ has beans} \Rightarrow \text{beans}}{tdo \textit{ has beans} \Rightarrow [tdo]\text{beans}} & \hspace{1cm} [ ]R \\
\frac{fp \textit{ has $}, tdo \textit{ has beans} \Rightarrow [fp]$ \otimes [tdo]\text{beans}}{\text{see above} \Rightarrow \text{Id}} & \hspace{1cm} \otimes R
\end{align*}
\]

$\Delta, fp \textit{ has $}, tdo \textit{ has beans}, [fp]$ \otimes [tdo]\textit{beans} \Rightarrow \{[fp]\textit{coffee} \otimes [tdo]\}$ $\Rightarrow C \textit{ lax} \hspace{1cm} \Rightarrow L$
From Axioms to Inference Rules

Focusing allows us to turn axioms such as

\[
\text{buy} : \left[ \text{fp} \right] \otimes \left[ \text{tdo} \right] \text{beans} \rightarrow \left\{ \left[ \text{fp} \right] \text{coffee} \otimes \left[ \text{tdo} \right] \right\}
\]

into a complete set of derived inference rules such as

\[
\begin{align*}
\Delta, \text{fp has coffee}, \text{tdo has } & \text{beans} \quad \Rightarrow \quad C \text{ lax} \quad \text{buy} \\
\Delta, \text{fp has } & \text{beans} \quad \Rightarrow \quad C \text{ lax}
\end{align*}
\]

Aside: to get this specific rule, some assumption on \( K \)'s possessions and other axioms are necessary

- No axioms with “head” \( $ \)
- Possessions are of the form \( K \textbf{ has } P \) for atoms \( P \)

The lax modality allows for somewhat stricter proof control than just focusing
Example Revisited: Deleting a File

\[
\langle K \rangle \text{do}(K, \text{create}) \\
\otimes \langle fs \rangle \text{may}(K, \text{create}) \\
\rightarrow \{ \exists f. \exists v. \\
\quad \neg \langle fs \rangle \text{owns}(K, f) \\
\otimes [fs] \text{current}(f, v) \\
\otimes [fs] \text{contains}(f, v, "") \\
\otimes [K] \text{contains}(f, v, "") \}
\]

- To explain: knowledge \[[K]A\] and persistent truth \(!A\)
- Following our judgmental approach, we add new form of assumptions
Persistent Assumptions

- Sequents have form

\[ \Gamma; \Delta \implies \gamma \]

where

- Persistent ants.
  \[ \Gamma ::= \bullet \mid \Gamma, A \text{ pers} \mid \Gamma, K \text{ knows } A \]

- Linear ants.
  \[ \Delta ::= \bullet \mid \Delta, A \text{ res} \mid \Delta, K \text{ has } A \]

- Succedents
  \[ \gamma ::= A \text{ true } \mid A \text{ lax} \]

- Persistent assumptions grow monotonically in bottom-up proof construction

- All present rules are updated to propagate \( \Gamma \) to all premises
Persistent truths can be used

\[
A \text{ pers} \in \Gamma, \Gamma; \Delta, A \text{ res} \Rightarrow \gamma \quad \overset{\text{pers}_L}{\Rightarrow} \quad \Gamma; \Delta \Rightarrow \gamma
\]

Truths whose proof requires no consumable resources are persistent

\[
\begin{bmatrix}
\Gamma; \bullet \Rightarrow A \text{ true} \\
\Gamma; \bullet \Rightarrow A \text{ pers}
\end{bmatrix} \quad \overset{\text{pers}_R}{\Rightarrow} \quad \quad \quad
\]
No new identity principle

\[
\begin{align*}
A \text{ pers}; A \text{ res} & \implies A \text{ true} \\
A \text{ pers}; \bullet & \implies A \text{ true} \\
A \text{ pers}; \bullet & \implies A \text{ pers}
\end{align*}
\]

New derived cut principle

\[
\begin{align*}
\Gamma; \bullet & \implies A \text{ true} \\
\Gamma; \bullet & \implies A \text{ pers} \\
\Gamma, A \text{ pers}; \Delta & \implies \gamma \\
\Gamma; \Delta & \implies \gamma
\end{align*}
\]
The Exponential Modality of Linear Logic

\[
\frac{\Gamma; \bullet \rightarrow A \text{ true}}{\Gamma; \bullet \rightarrow !A \text{ true}} \quad !R \quad \frac{\Gamma, A \text{ pers}; \Delta \rightarrow \gamma}{\Gamma; \Delta, !A \text{ res} \rightarrow \gamma} \quad !L
\]

- Internalize persistent truth
- Identity expansion and cut reduction work easily
A Judgment of Knowledge

- **K knows A** \(\sim\) knowledge as persistent possession
- Persistent knowledge can be used by **K**

\[
\begin{align*}
K \text{ knows } A \in \Gamma & \quad \Gamma; \Delta, A \text{ res } \implies \gamma \\
\Gamma; \Delta & \implies \gamma
\end{align*}
\]

- Truth whose proofs require only local knowledge can be known

\[
\left[ \begin{array}{c}
\Gamma|_K; \bullet \implies A \\
\Gamma; \bullet \implies K \text{ knows } A
\end{array} \right] \implies \text{knows}_R
\]

- \(\Gamma|_K\) restricts to antecedents of the form **K knows**
Cut and Identity for Knowledge

- No new identity principle

\[
K \text{ knows } A; A \text{ res } \Rightarrow A \text{ true } \\
\frac{\text{id}}{K \text{ knows } A; \bullet \Rightarrow A \text{ true}} \\
\frac{\text{knows}_L}{K \text{ knows } A; \bullet \Rightarrow K \text{ knows } A}
\]

- New derived cut principle

\[
\frac{\text{id}}{K \text{ knows } A; \bullet \Rightarrow A \text{ true}} \\
\frac{\text{knows}_R}{\Gamma; \bullet \Rightarrow K \text{ knows } A} \\
\frac{\text{cut}_{\text{knows}}}{\Gamma, K \text{ knows } A; \Delta \Rightarrow \gamma}
\]

\[
\frac{\text{id}}{\Gamma; \Delta \Rightarrow \gamma}
\]
Knowledge as a Modality

\[ \frac{\Gamma|_K; \bullet \Rightarrow A \text{ true}}{\Gamma; \bullet \Rightarrow [K]A \text{ true}} \] \hspace{2cm} \[ \frac{\Gamma, K \text{ knows } A; \Delta \Rightarrow \gamma}{\Gamma; \Delta, [K]A \Rightarrow \gamma} \]

- Identity expansion and cut reduction as usual
- Knowledge is like indexed judgmental S4
Outline

1 Background: Proof-Carrying Authorization
2 Logical Foundations
   1 Resources (linear logic)
   2 Possessions (linear epistemic logic)
   3 Effects (linear lax logic)
   4 From axioms to inference rules via focusing
   5 Persistent truth and knowledge (epistemic logic)
3 Policy Consequences
   1 State invariants
   2 Proving metatheorems
4 Speculation: linear epistemic logic programming
Characterizing State

- Need to characterize the system states so we can reason about the policy
- System states are pairs $\Gamma; \Delta$
  - $\Gamma$ is persistent
  - $\Delta$ is linear
  - We do not care about the right-hand side, but it must have the form $C \text{lax}$ to permit effects
- Using this characterization, we turn each semantics rule into (one or more) rewrite rules for system states
- Using the rewrite rules we can prove theorems about the semantics
Example: Characterizing File System State

- Each persistent judgment in $\Gamma$ is one of
  - A policy rule or semantics action
  - $fs$ knows contents($F, V, S$) or $K$ knows contents($F, V, S$)
  - $\langle fs\rangle$user($K$) or $\langle fs\rangle$owns($K, F$)

- Each linear judgment in $\Delta$ is one of
  - $fs$ has current($F, V$)
  - $\langle K\rangle$do($K, A$)

- For each file $F$, there is at most one $V$ such that $fs$ has current($F, V$)
Example: Reading a File

• Specification

\[ \langle K \rangle \text{do}(K, \text{on}(F, \text{delete})) \]
\[ \otimes \, \langle fs \rangle \text{may}(K, \text{on}(F, \text{delete})) \]
\[ \otimes \, [fs]\text{current}(F, V) \]
\[ \rightarrow \{1\} \]

• Rewrite step

\[ \Gamma; \Delta, \langle K \rangle \text{do}(K, \text{on}(F, \text{delete})), \text{fs has current}(F, V) \]
\[ \rightarrow \Gamma; \Delta \]

provided \( \Gamma \vdash \langle fs \rangle \text{may}(K, \text{on}(F, \text{delete})) \)
Example: Writing to a File

- **Specification**

  \[
  \langle K \rangle \text{do}(K, \text{on}(F, \text{write}(S)))
  \]

- **Rewrite interpretation**

  \[
  \Gamma; \Delta, \langle K \rangle \text{do}(K, \text{on}(F, \text{write}(S))), \text{fs has current}(F, V) \\
  \rightarrow \Gamma, \text{fs knows contains}(F, v', S), K \text{ knows contains}(F, v', S); \Delta, \text{fs has current}(F, v')
  \]

  for a new \( v' \) provided \( \Gamma \vdash \langle \text{fs} \rangle \text{may}(K, \text{on}(F, \text{write}(S))) \)
Theorem (Knowledge Safety)

If $\Gamma; \Delta$ is a file system state such that $\Gamma; \Delta \rightarrow \Gamma', K$ knows contents($F, V, S$); $\Delta'$

then either $K$ knows contents($F, T, S$) $\in \Gamma$ or the step was a create, read, or write action $A$ on $F$ by $K$ permitted by the policy (as evidenced by a proof of $\langle fs \rangle \text{may}(K, A)$)

Proof.

By case analysis of the possible rewrite step schemata.
Stratification

- The proofs still apply as long as the signed policy statements do not involve any effects or possessions.
- In general, the system should be stratified so proofs of authorization are effect-free.
  - Uses of authorizations are the effect.
  - Linear theorem proving of authorization theorem does not consume the certificates!
- Located certificates and proofs:
  - File system example abstract away from location of proofs.
  - Could specify client of server to produce the proof.
Another Example: Electronic Voting

va = voting authority

\langle va \rangle \text{hasvote}(K) \quad \text{(linear certificate)}
\otimes !\langle va \rangle \text{candidate}(L) \quad \text{(persistent certificate)}
\otimes [K] \langle K \rangle \text{votefor}(L) \quad \text{(linear possession of cert.)}
\otimes [va] \text{voting} \quad \text{(linear “token”)}
\otimes [va] \text{votecount}(N) \quad \text{(linear “token”)}
\rightarrow \{ [va] \text{vote}(L) \quad \text{(linear vote result)}
\otimes [va] \text{count}(N + 1)
\otimes [va] \text{voting} \}$
Example: Counting Electronic Votes

\[ [va] \text{voting} \quad \text{(linear “token”) \ (linear trigger)} \]
\[ \otimes \langle va \rangle \text{pollclosed} \quad \text{linear “token”) \ (linear trigger)} \]
\[ \otimes [va] \text{votecount}(N) \quad \text{(linear “token”) \ (new token)} \]
\[ \rightarrow \{ [va] \text{counting}(N) \} \]

\[ [va] \text{counting}(0) \quad \text{(vote counting done)} \]
\[ \rightarrow \{ [va] \text{done} \} \]

\[ [va] \text{counting}(N) \otimes !N > 0 \quad \text{(token and condition)} \]
\[ \otimes [va] \text{votefor}(L) \quad \text{(vote for \( L \), being tallied)} \]
\[ \otimes [va] \text{numvotes}(L, K) \quad \text{(vote counter)} \]
\[ \rightarrow \{ [va] \text{counting}(N) \}
\[ \quad \otimes [va] \text{votes}(L, K + 1) \} \]
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Speculation: Linear Epistemic Logic Programming

- Idea: Give a forward chaining ("bottom-up") logic programming interpretation as a distributed programming language
- By design, the implementation will satisfy the specification
- By design, the implementation will satisfy the theorems proven about the specification
- Based on the polarized, focusing interpretation
  - Some additional restrictions will be necessary
  - Mode checking, staging verification, . . .
- Must execute protocols on multiple hosts
Example: A Binary Counter

- State invariants for each principal (= bit) $K$
  - For each $K$, either $K$ knows next($L$) or $K$ knows last
  - For each $K$, either $K$ has zero or $K$ has one
  - For one $K$, $K$ has inc may be present

- Program

\[
[K]\text{inc} \otimes [K]\text{zero} \leadsto \{[K]\text{one}\}
\]
\[
[K]\text{inc} \otimes [K]\text{one} \otimes [K]\text{next}(L) \leadsto \{[K]\text{zero} \otimes [L]\text{inc}\}
\]
\[
[K]\text{inc} \otimes [K]\text{one} \otimes [K]\text{last} \leadsto \{[K]\text{zero}\}
\]

- Have hand-compiled version in Meld on “blinky-blocks”
Atomicity

- In general, complex multi-party contract signing protocols may be necessary to ensure atomicity of the rules.
- Example (with conditions from two parties)
  \[
  \text{prev}(K) \otimes [K]\text{carry} \otimes [L]\text{zero} \rightarrow \{[L]\text{one}\}
  \]
  \[
  \text{prev}(K) \otimes [K]\text{carry} \otimes [L]\text{one} \rightarrow \{[L]\text{zero} \otimes [L]\text{carry}\}
  \]
- Inference system suggests “truth” as a trusted third party that leaks no information.
- Looking for a suitable lower-level calculus to compile to for expressing communication protocols.
Goals

- Logical specification of distributed authorization policies
- Reliable enforcement of such high-level policies (PCA)
  - Implemented in practical proof-carrying file system
- Mechanized reasoning about consequences of policies:
  - Evolution of system state
  - Principals’ knowledge (information)
  - Principals’ possessions (consumable resources)

Approach: linear epistemic logic

- Pedantic definition from judgmental principles
- Possession is linear knowledge
- Specification at extremely high level of abstraction
Ongoing and Future Work

- Define distributed forward chaining linear epistemic logic programming language
- Compile to distributed code executing multi-party communication protocols
- Prove correctness with respect to rewriting semantics
  - Atomicity of rules most difficult
  - Identify tractable language subset
  - Eliminate some uses of trusted third party (\(=\) truth)
- Mechanize reasoning about policies
  - “See” my talk at LFMTP yesterday
For More . . .


- Further pointers from this workshop, I hope!