Proof-Carrying Code in a Session-Typed Process Calculus

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Why do we trust software?

We don't!

To the extent that we do, we rely on:

- Digital signatures (state-of-the-practice)
- Formal proof (state-of-the-art)

Can we combine digital signatures and proofs?

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Communicating processes

- Name-passing (mobile)
- Value-passing (applied)
- Proof-passing (proof-carrying)

Reason about process behavior

- Deadlock-freedom
- Session fidelity
- Termination

Reason about values and proofs

- Types
- Correctness of proofs
- Validity of signatures
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System Modeling and Security Properties

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  - Types
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Approach

Communicating processes
Value-passing extension of $\pi$-calculus
Reason about process behavior

[CONCUR'10]

Session types
Curry-Howard isomorphism between Intuitionistic linear propositions and session types
Sequent proofs and $\pi$-calculus processes
Proof reduction and process reduction
Reason about values and proofs

Dependent session types
[PPDP'11]
Terms and proofs from dependent type theory
Add proof irrelevance (to avoid sending proofs)
Add affirmation (to capture digital signatures)
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- Reason about values and proofs
  - Dependent session types [PPDP’11]
  - Terms and proofs from dependent type theory
  - Add proof irrelevance (to avoid sending proofs)
  - Add affirmation (to capture digital signatures)
1. Session types for $\pi$-calculus
2. Dependent session types
3. Proof irrelevance
4. Affirmation and digital signatures
5. Conclusion
Session types: judgment forms

- Judgment $P :: x : A$
  - Process $P$ offers service $A$ along channel $x$
- Linear sequent

$$x_1 : A_1, \ldots, x_n : A_n \Rightarrow P :: x : A$$

$P$ uses $x_i : A_i$ and offers $x : A$.
- Cut as composition

$$\Delta \Rightarrow A \quad \Delta', \quad A \Rightarrow C$$
cut

$$\Delta, \Delta' \Rightarrow C$$
- Identity as forwarding

$$A \Rightarrow \text{id}$$
Session types: judgment forms

- Judgment $P :: x : A$
  - Process $P$ offers service $A$ along channel $x$
- Linear sequent
  \[
  x_1 : A_1, \ldots, x_n : A_n \Rightarrow P :: x : A
  \]
  $\Delta$

  $P$ uses $x_i : A_i$ and offers $x : A$.
- Cut as composition
  \[
  \Delta \Rightarrow x : A \quad \Delta', x : A \Rightarrow z : C
  \]
  $\Delta, \Delta' \Rightarrow z : C$
  cut

- Identity as forwarding
  \[
  A \Rightarrow \quad \text{id}
  \]
  $A$
Session types: judgment forms

- Judgment $P :: x : A$
  - Process $P$ offers service $A$ along channel $x$
- Linear sequent
  \[
  \frac{x_1 : A_1, \ldots, x_n : A_n}{\Delta} \Rightarrow P :: x : A
  \]
  $P$ uses $x_i : A_i$ and offers $x : A$.
- Cut as composition
  \[
  \frac{\Delta \Rightarrow P :: x : A \quad \Delta', x : A \Rightarrow Q :: z : C}{\Delta, \Delta' \Rightarrow (\nu x)(P \mid Q) :: z : C}
  \] cut
- Identity as forwarding
  \[
  A \Rightarrow A \quad \text{id}
  \]
Session types: judgment forms

- Judgment $P :: x : A$
  - Process $P$ offers service $A$ along channel $x$
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  \[
  x_1 : A_1, \ldots, x_n : A_n \Rightarrow P :: x : A
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  \[
  \Delta, \Delta' \Rightarrow (\nu x)(P \mid Q) :: z : C
  \]
  cut
- Identity as forwarding
  \[
  x : A \Rightarrow z : A
  \]
  id
Session types: judgment forms

- Judgment \( P :: x : A \)
  - Process \( P \) offers service \( A \) along channel \( x \)

- Linear sequent

\[
\begin{align*}
\Delta & \vdash P :: x : A \\
\Delta_1 : \ldots : \Delta_n & \vdash x_1 : A_1, \ldots, x_n : A_n \\
\Delta & \vdash \Delta, \Delta' \Rightarrow (\nu x)(P \mid Q) :: z : C
\end{align*}
\]

\( P \) uses \( x_i : A_i \) and offers \( x : A \).

- Cut as composition

\[
\Delta \Rightarrow P :: x : A \quad \Delta', x : A \Rightarrow Q :: z : C
\]

\[
\Delta, \Delta' \Rightarrow (\nu x)(P \mid Q) :: z : C
\]

- Identity as forwarding

\[
\begin{align*}
\Rightarrow [x \leftrightarrow z] :: z : A \\
\text{id}
\end{align*}
\]
Session types: input ($A \rightarrow B$)

- $P ::= x : A \rightarrow B$
  - $P$ inputs an $A$ along $x$ and then behaves as $B$
- Right rule: offer of service

$$\Delta, \quad A \Rightarrow \quad B \quad \Delta \Rightarrow \quad A \rightarrow B \quad \rightarrow R$$

- Can reuse $x$, due to linearity
- Left rule: matching use of service

$$\Delta \Rightarrow \quad A \quad \Delta', \quad B \Rightarrow \quad C \quad \Delta, \Delta', \quad A \rightarrow B \Rightarrow \quad C \quad \rightarrow L$$

- Can reuse $x$, due to linearity
- Channel $y$ must be new
Session types: input \((A \multimap B)\)

- \(P :: x : A \multimap B\)
  - \(P\) inputs an \(A\) along \(x\) and then behaves as \(B\)
- Right rule: offer of service
  \[
  \begin{align*}
  \Delta, y : A & \Rightarrow x : B \\
  \Delta & \Rightarrow x : A \multimap B \\
  \end{align*}
  \]
  - Can reuse \(x\), due to linearity
- Left rule: matching use of service
  \[
  \begin{align*}
  \Delta & \Rightarrow A, \Delta', B \Rightarrow C \\
  \Delta, \Delta', A \multimap B & \Rightarrow C \\
  \end{align*}
  \]
  - Can reuse \(x\), due to linearity
  - Channel \(y\) must be new
Session types: input $(A \multimap B)$

- $P :: x : A \multimap B$
  - $P$ inputs an $A$ along $x$ and then behaves as $B$
  - Right rule: offer of service
    \[
    \begin{align*}
    \Delta, y:A & \Rightarrow P :: x : B \\
    \Delta & \Rightarrow x(y).P :: x : A \multimap B
    \end{align*}
    \]
    - Can reuse $x$, due to linearity
  - Left rule: matching use of service
    \[
    \begin{align*}
    \Delta & \Rightarrow A \quad \Delta', B \Rightarrow C \\
    \Delta, \Delta', A \multimap B & \Rightarrow C
    \end{align*}
    \]
    - Can reuse $x$, due to linearity
    - Channel $y$ must be new
Session types: input \((A \rightarrow B)\)

- \(P :: x : A \rightarrow B\)
  - \(P\) inputs an \(A\) along \(x\) and then behaves as \(B\)
- Right rule: offer of service

\[
\Delta, y : A \Rightarrow P :: x : B \\
\Delta \Rightarrow x(y).P :: x : A \rightarrow B
\]

  - Can reuse \(x\), due to linearity
- Left rule: matching use of service

\[
\Delta \Rightarrow y : A \quad \Delta', x : B \Rightarrow z : C \\
\Delta, \Delta', x : A \rightarrow B \Rightarrow \\
\Delta, \Delta', x : A \rightarrow B \Rightarrow z : C
\]

  - Can reuse \(x\), due to linearity
  - Channel \(y\) must be new
Session types: input \((A \to B)\)

- \(P :: x : A \to B\)
  - \(P\) inputs an \(A\) along \(x\) and then behaves as \(B\)
- Right rule: offer of service
  \[
  \Delta, y:A \Rightarrow P :: x : B \quad \Delta \Rightarrow x(y).P :: x : A \to B \quad \to R
  \]
  - Can reuse \(x\), due to linearity
- Left rule: matching use of service
  \[
  \Delta \Rightarrow P :: y : A \quad \Delta', x:B \Rightarrow Q :: z : C
  \[
  \Delta, \Delta', x:A \to B \Rightarrow (\nu y)x\langle y \rangle.(P | Q) :: z : C \quad \to L
  \]
  - Can reuse \(x\), due to linearity
  - Channel \(y\) must be new
Session types: reduction

- Proof reduction

\[
\frac{\Delta, A \Rightarrow B}{\Delta \Rightarrow A \rightarrow B} \quad \frac{\Delta_1 \Rightarrow A \quad \Delta_2, B \Rightarrow C}{\Delta_1, \Delta_2, A \rightarrow B \Rightarrow C} \quad \frac{\Delta_1 \Rightarrow A}{\Delta, \Delta_1, \Delta_2 \Rightarrow C}
\]

- Corresponding process reduction

\[
\Delta, \Delta_1, \Delta_2 \Rightarrow (\nu x)(x(y).P_1 \mid (\nu w)(x\langle w \rangle.(P_2 \mid Q)))) :: z : C
\]

\[
\Delta, \Delta_1, \Delta_2 \Rightarrow (\nu x)((\nu w)(P_2 \mid P_1\{w/y\}) \mid Q) :: z : C
\]
Session types: other connectives

- Linear propositions as session types
  
  $P :: x : A \rightarrow B$  Input a $y : A$ along $x$ and behave as $B$
  
  $P :: x : A \otimes B$  Output a new $y : A$ along $x$ and behave as $B$
  
  $P :: x : 1$  Terminate session on $x$
  
  $P :: x : A \& B$  Offer choice between $A$ and $B$ along $x$
  
  $P :: x : A \oplus B$  Offer either $A$ or $B$ along $x$
  
  $P :: x : !A$  Offer $A$ persistently along $x$

- Sequent proofs as process expressions

- Proof reduction as process reduction
Two small examples

- PDF indexing service, version 1

  \[ \text{index}_1 : !(\text{file} \rightarrow \text{file} \otimes 1) \]

  Persistently offer to input a file, then output a file and terminate session. Intent: input PDF, output indexed PDF for keyword search.

- Persistent file storage

  \[ \text{store}_1 : !(\text{file} \rightarrow !(\text{file} \otimes 1)) \]

  Persistently offer to input a file, then output a persistent handle for retrieving this file. Intent: output file is the same as input file.
1. Session types for $\pi$-calculus
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Term passing

- Types $\tau$ from a (dependent) type theory
- Hypothetical judgment $\Gamma \vdash M : \tau$

Some example type constructors

- $\Pi x : \tau . \sigma$, $\tau \rightarrow \sigma$ Functions from $\tau$ to $\sigma$
- $\Sigma x : \tau . \sigma$, $\tau \times \sigma$ Pairs of a $\tau$ and a $\sigma$
- nat Natural numbers

Integrate into sequent calculus

$\Psi ; \Gamma ; \Delta \Rightarrow P :: x : A$

<table>
<thead>
<tr>
<th>$\Psi$</th>
<th>$\Gamma$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>term variables</td>
<td>persistent channels</td>
<td>linear channels</td>
</tr>
</tbody>
</table>

linear
Term passing: input \((\forall y:\tau.A)\)

- \(P :: x : \forall y:\tau A\)
  - \(P\) inputs an \(M : \tau\) along \(x\) and then behaves as \(A\{M/x\}\)
- Right rule: offer of service
  \[
  \frac{\Psi, y:\tau ; \Gamma ; \Delta \Rightarrow A}{\Psi ; \Gamma ; \Delta \Rightarrow \forall y:\tau.A} \quad \forall R
  \]

- Left rule: matching use of service
  \[
  \frac{\Psi \vdash M : \tau \quad \Psi ; \Gamma ; \Delta', A\{M/y\} \Rightarrow C}{\Psi ; \Gamma ; \Delta', \forall y:\tau.A \Rightarrow C} \quad \forall L
  \]

- Proof reduction yields
  \[
  \rightarrow
  \]
Term passing: input \((\forall y:\tau.A)\)

- \(P :: x : \forall y:\tau A\)
  - \(P\) inputs an \(M : \tau\) along \(x\) and then behaves as \(A\{M/x\}\)
- Right rule: offer of service
  \[
  \begin{align*}
  \Psi, y:\tau ; \Gamma ; \Delta &\Rightarrow x : A \\
  \Psi ; \Gamma ; \Delta &\Rightarrow x : \forall y:\tau.A
  \end{align*}
  \]
- Left rule: matching use of service
  \[
  \begin{align*}
  \Psi \vdash M : \tau &\quad \Psi ; \Gamma ; \Delta', A\{M/y\} \Rightarrow C \\
  \Psi ; \Gamma ; \Delta', \forall y:\tau.A &\Rightarrow C
  \end{align*}
  \]
- Proof reduction yields
  \[
  \rightarrow
  \]
Term passing: input ($\forall y: \tau. A$)

- $P :: x : \forall y: \tau A$
  - $P$ inputs an $M : \tau$ along $x$ and then behaves as $A\{M/x\}$
- Right rule: offer of service

$$\Psi, y: \tau ; \Gamma ; \Delta \Rightarrow P :: x : A$$

$$\Psi ; \Gamma ; \Delta \Rightarrow x(y). P :: x : \forall y: \tau. A$$

- Left rule: matching use of service

$$\Psi \vdash M : \tau \quad \Psi ; \Gamma ; \Delta', A\{M/y\} \Rightarrow C$$

$$\Psi ; \Gamma ; \Delta', \forall y: \tau. A \Rightarrow C$$

- Proof reduction yields

$$\rightarrow$$
Term passing: input (\(\forall y:\tau.A\))

- \(P :: x : \forall y:\tau A\)
  - \(P\) inputs an \(M : \tau\) along \(x\) and then behaves as \(A\{M/x\}\)

- Right rule: offer of service

\[
\begin{align*}
\psi, y:\tau ; \Gamma ; \Delta & \Rightarrow P :: x : A \\
\psi ; \Gamma ; \Delta & \Rightarrow x(y).P :: x : \forall y:\tau.A
\end{align*}
\]

- Left rule: matching use of service

\[
\begin{align*}
\psi \vdash M : \tau & \quad \psi ; \Gamma ; \Delta', x:A\{M/y\} \Rightarrow z : C \\
\psi ; \Gamma ; \Delta', x:\forall y:\tau.A & \Rightarrow z : C
\end{align*}
\]

- Proof reduction yields
Term passing: input $(\forall y:\tau. A)$

- $P :: x : \forall y:\tau A$
  - $P$ inputs an $M : \tau$ along $x$ and then behaves as $A\{M/x\}$
- Right rule: offer of service
  \[
  \frac{\Psi, y:\tau ; \Gamma ; \Delta \Rightarrow P :: x : A}{\Psi ; \Gamma ; \Delta \Rightarrow x(y).P :: x : \forall y:\tau A} \quad \forall R
  \]
- Left rule: matching use of service
  \[
  \frac{\Psi \vdash M : \tau \quad \Psi ; \Gamma ; \Delta', x:A\{M/y\} \Rightarrow Q :: z : C}{\Psi ; \Gamma ; \Delta', x:\forall y:\tau.A \Rightarrow x\langle M\rangle.Q :: z : C} \quad \forall L
  \]
- Proof reduction yields
Term passing: input \((\forall y:\tau. A)\)

- \(P :: x : \forall y:\tau A\)
  - \(P\) inputs an \(M : \tau\) along \(x\) and then behaves as \(A{M/x}\)
- Right rule: offer of service
  \[
  \frac{\Psi, y:\tau ; \Gamma ; \Delta \Rightarrow P :: x : A}{\Psi ; \Gamma ; \Delta \Rightarrow x(y).P :: x : \forall y:\tau.A}
  \]
  \(\forall R\)

- Left rule: matching use of service
  \[
  \frac{\Psi \vdash M : \tau \quad \Psi ; \Gamma ; \Delta', x:A{M/y} \Rightarrow Q :: z : C}{\Psi ; \Gamma ; \Delta', x:\forall y:\tau.A \Rightarrow x\langle M\rangle.Q :: z : C}
  \]
  \(\forall L\)

- Proof reduction yields
  \[
  (\nu x)(x(y).P \mid x\langle M\rangle.Q) \rightarrow
  \]
Term passing: input ($\forall y:\tau. A$)

- $P :: x : \forall y:\tau A$
  - $P$ inputs an $M : \tau$ along $x$ and then behaves as $A\{M/x\}$

- Right rule: offer of service

\[
\frac{\Psi, y:\tau ; \Gamma ; \Delta \Rightarrow P :: x : A}{\Psi ; \Gamma ; \Delta \Rightarrow x(y).P :: x : \forall y:\tau.A} \quad \forall R
\]

- Left rule: matching use of service

\[
\frac{\Psi \vdash M : \tau \quad \Psi ; \Gamma ; \Delta', x:A\{M/y\} \Rightarrow Q :: z : C}{\Psi ; \Gamma ; \Delta', x:\forall y:\tau.A \Rightarrow x\langle M\rangle.Q :: z : C} \quad \forall L
\]

- Proof reduction yields

\[
(\nu x)(x(y).P \mid x\langle M\rangle.Q) \rightarrow (\nu x)(P\{M/y\} \mid Q)
\]
Term passing: other connectives

- Quantified proposition as dependent session types
  
  \[ x : \forall y: \tau. A \quad \text{Input an } M : A \text{ along } x \text{ and behave as } A\{M/y\} \]
  
  \[ x : \$\tau \to A \quad \text{Input an } M : A \text{ along } x \text{ and behave as } A \]
  
  \[ x : \exists y: \tau. A \quad \text{Output an } M : A \text{ along } x \text{ and behave as } A\{M/y\} \]
  
  \[ x : \$\tau \otimes A \quad \text{Output an } M : A \text{ along } x \text{ and behave as } A \]

- \$\tau \to A \text{ as shorthand for } \forall y: \tau. A \text{ if } y \text{ not free in } A

- \$\tau \otimes A \text{ as shorthand for } \exists y: \tau. A \text{ if } y \text{ not free in } A

- We will omit the ‘$’ for readability
Examples, carrying proofs

PDF indexing service

\[
\text{index}_1 : ! (\text{file} \rightarrow \text{file} \otimes 1)
\]
\[
\text{index}_2 : ! (\forall f : \text{file}. \text{pdf}(f) \rightarrow \exists g : \text{file}. \text{pdf}(g) \otimes 1)
\]

Persistently offer to input a file \( f \), a proof that \( f \) is in PDF format, then output a PDF file \( g \), and a proof that \( g \) is in PDF format and terminate the session.

Persistent file storage

\[
\text{store}_1 : ! (\text{file} \rightarrow ! (\text{file} \otimes 1))
\]
\[
\text{store}_2 : ! (\forall f : \text{file}. \exists g : \text{file}. g \simeq f \otimes 1)
\]

Persistently offer to input a file, then output a persistent channel for retrieving this file and a proof that the two are equal.
Outline

1. Session types for $\pi$-calculus
2. Dependent session types
3. Proof irrelevance
4. Affirmation and digital signatures
5. Conclusion
Proof irrelevance

- In many examples, we want to know that proofs exist, but we do not want to transmit them
  - We can easily check pdf(g) when using the indexing service
  - The proof of $g \vdash f$ (by reflexivity) would not be informative
- Use proof irrelevance in type theory
- $M : [\tau] \to M$ is a term of type $\tau$ that is computationally irrelevant
Proof irrelevance: rules

- **Introduction and elimination**

\[
\frac{\psi^\oplus \vdash M : \tau}{\psi \vdash [M] : [\tau]} \quad []I \quad \frac{\psi \vdash M : [\tau] \quad \psi, x : \tau \vdash N : \sigma}{\psi \vdash \text{let } [x] = M \text{ in } N : \sigma} \quad []E
\]

- \(\psi^\oplus\) promotes hypotheses \(x : \tau\) to \(x : \tau\)

- In examples, may use pattern matching instead of \textbf{let}

- By agreement, terms \([M]\) will be erased before transmission

- Typing guarantees this can be done consistently
Examples with proof irrelevance

- Mark proofs as computationally irrelevant
- PDF indexing service

\[
\text{index}_2 : \neg(\forall f: \text{file. } \text{pdf}(f) \implies \exists g: \text{file. } \text{pdf}(g) \otimes 1)
\]
\[
\text{index}_3 : \neg(\forall f: \text{file. } [\text{pdf}(f)] \implies \exists g: \text{file. } [\text{pdf}(g)] \otimes 1)
\]

- Persistent file storage

\[
\text{store}_2 : \neg(\forall f: \text{file. } !\exists g: \text{file. } g \vDash f \otimes 1)
\]
\[
\text{store}_3 : \neg(\forall f: \text{file. } !\exists g: \text{file. } [g \vDash f] \otimes 1)
\]

- After erasure, communication can be optimized further
Examples: affirming the existence of proofs

- In the PDF indexing example, we may want to have some evidence that $g$ and $f$ agree.

\[ \text{index}_4 : \text{!}(\forall f:\text{file. [pdf}(f)\text{]} \land g:\text{file. [pdf}(g)\text{]} \otimes [\text{agree}(g, f)] \otimes 1) \]

agree($g, f$) if $g$ and $f$ differ at most in the index

- Since no proof is transmitted, client may require indexer $X$’s explicit affirmation (= digital signature)!

- Similarly, in the persistent file storage example
Outline

1. Session types for $\pi$-calculus
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5. Conclusion
Affirmation

- Judgment $M : K \tau$
  - Principal K affirms property $\tau$ due to evidence $M$.
    \[
    \begin{array}{c}
    \Psi \vdash M : \tau \\
    \Psi \vdash \langle M : \tau \rangle_K : K \tau
    \end{array}
    \]
    (affirms)

- Internalize judgment as proposition $\diamond_{K\tau}$
  \[
  \begin{array}{c}
  \Psi \vdash M : K \tau \\
  \Psi \vdash M : \diamond_{K\tau}
  \end{array}
  \]

- Note same principal $K$ in premises and conclusion of $\diamond E$
- $\langle M : \tau \rangle_K$ can be realized by $K$’s signature on $M : \tau$
- Assume some public key infrastructure
- $\diamond_K$ is a $K$-indexed family of strong monads
Examples: affirmations

- PDF indexing service, with indexer $X$

\[
\text{index}_5 : \text{!}(\forall f:\text{file.} [\text{pdf}(f)]) \\
\quad \rightarrow \exists g:\text{file.} [\text{pdf}(g)] \otimes \text{\Diamond}_X[\text{agree}(g, f)] \otimes \text{1)}
\]

- Persistent file storage, with file system $Y$

\[
\text{store}_4 : \text{!}(\forall f:\text{file.} \text{!}\exists g:\text{file.} \text{\Diamond}_Y[g \overset{\cdot}{=} f] \otimes \text{1})
\]

- Idiom $\text{\Diamond}_K[\tau]$ may transmit
  - $\langle [\tau] \rangle_K$, a certificate, digitally signed by $K$ affirming $\tau$
  - Some proof that $[\tau]$ follows from affirmations by $K$, according to the laws of $\text{\Diamond}_K$
Example: a PDF compression service

- A PDF compression service, with compressor $C$

  \[
  \text{compress} : (\forall f:\text{file. } [\text{pdf}(f)] \quad \leadsto \quad \exists g:\text{file. } [\text{pdf}(g)] \otimes \Diamond C[\text{approx}(g, f)] \otimes 1)
  \]

- A consolidator service: indexing and compression

  \[
  \text{ixc} : (\forall f:\text{file. } [\text{pdf}(f)] \quad \leadsto \quad \exists g:\text{file. } [\text{pdf}(g)] \otimes \Diamond X \Diamond C[\text{approx}(g, f)] \otimes 1)
  \]

- Have to trust both $X$ and $C$!
Example: consolidator implementation

- **Specification**

  \[\text{ixc} : \forall f : \text{file. } [\text{pdf}(f)] \rightarrow \exists g : \text{file. } [\text{pdf}(g)] \otimes \diamond X \diamond C[\text{approx}(g, f)] \otimes 1\]

- **Implementation**

  \[\text{consolidator} = !\text{ixc}(a).a(f_1).a([p_1]).\]

  \[(\nu b)\text{index}\langle b \rangle . b\langle f_1 \rangle . b\langle [p_1] \rangle . b(f_2) . b([p_2]) . b(q_2).\]

  \[(\nu c)\text{compress}\langle c \rangle . c\langle f_2 \rangle . c\langle [p_2] \rangle . c(f_3) . c([p_3]) . c(q_3).\]

  \[a\langle f_3 \rangle . a\langle [p_3] \rangle . a\langle \text{comb } q_2 q_3 \rangle . 0\]

- **Certificate types**

  \[q_2 : \diamond X [\text{agree}(f_2, f_1)]\]

  \[q_3 : \diamond C [\text{approx}(f_3, f_2)]\]

  \[\text{comb } q_2 q_3 : \diamond C \diamond X [\text{approx}(f_3, f_1)]\]
Certificate combination

- Certificate types

\[ q_2 : \Diamond_X [\text{agree}(f_2, f_1)] \]
\[ q_3 : \Diamond_C [\text{approx}(f_3, f_2)] \]
\[ \text{comb } q_2 \ q_3 : \Diamond_C \Diamond_X [\text{approx}(f_3, f_1)] \]

- Proof

\begin{align*}
\text{ida} & : \text{agree}(f_2, f_1) \rightarrow \text{approx}(f_2, f_1) \\
\text{tra} & : \text{approx}(f_3, f_2) \rightarrow \text{approx}(f_2, f_1) \rightarrow \text{approx}(f_3, f_1) \\
\text{comb } q_2 \ q_3 & = \\
& \text{let } \langle [q'_3] : [\text{approx}(f_3, f_2)] \rangle_c = q_3 \text{ in} \\
& \langle \text{let } \langle [q'_2] : [\text{agree}(f_2, f_1)] \rangle_x = q_2 \text{ in} \\
& \langle [\text{tra } q'_3 (\text{ida } q'_2)] : \_ \rangle_x : \_ \rangle_c
\end{align*}
Trust axioms

- Affirmations track aspects of provenance and info. flow
  - “Diamonds are forever”
  - In general, $\not\vdash \diamondsuit_K \tau \rightarrow \tau$
  - Need declassification

- Trust axioms
  - For specific types $\tau$ and principals $K$:
    \[
    \text{trust}_{K,\tau} : \diamondsuit_K \tau \rightarrow \tau
    \]
  - Implementable, in general, by stripping signature
  - Omitted proofs $[\tau]$ cannot be recovered, in general
    - $\not\vdash [\tau] \rightarrow \tau$ not implementable, in general
    - $\not\vdash \diamondsuit_K [\tau] \rightarrow \tau$ not implementable, in general
Example: mobile code

- For sensitive documents we want to run indexing locally
- Specification

\[ \text{index}_6 : ! (\bigtriangleup x (\forall f : \text{file. } [\text{pdf}(f)]) \rightarrow \Sigma g : \text{file. } [\text{pdf}(g)] \times [\text{agree}(g, f)]) \otimes 1) \]

- Service persistently offers a function for indexing
- Cannot leak information since only process layer can communicate
Signed certificates may have external meaning
Signed certificates may flow in both directions

\[ \text{index}^u_\gamma : !(\Diamond_u [\text{pay}(u, X, 1)] \]
\[ \quad \rightarrow (\forall f : \text{file.} [\text{pdf}(f)]) \]
\[ \quad \rightarrow \exists g : \text{file.} [\text{pdf}(g)] \otimes \Diamond_x [\text{agree}(g, f)] \otimes 1) \]

Need to make principals more explicit?
Some experience in proof-carrying authorization
1. Session types for $\pi$-calculus
2. Dependent session types
3. Proof irrelevance
4. Affirmation and digital signatures
5. Conclusion
Summary

- A Curry-Howard isomorphism
  - Linear propositions as session types
    \[ A \rightarrow B \] (input), \[ A \otimes B \] (output), \[ A \& B \] (external choice)
    \[ A \oplus B \] (internal choice), \[ !A \] (replication)
  - Sequent proofs as \( \pi \)-calculus processes
    with a binary guarded choice and channel forwarding
  - Cut reduction as \( \pi \)-calculus reduction
- Term-passing extension with a type theory
  - \( \forall x: \tau. A \) (term input), \( \exists x: \tau. A \) (term output)
- Additional type theory constructs
  - \([\tau]\) for proof irrelevance (not transmitted)
  - \(\Diamond_K \tau\) for affirmations (evidenced by digital signatures)
Assessment

- Strong basis in logic and type theory
  - Modular construction and extensibility
  - Integrated computation and reasoning
- Uniform logical integration
  - Proofs (implicit or explicit)
  - Affirmations (implicit or explicit signatures)
- Enable gradual integration of formal proofs in current practice based on digital signatures?
Current and Future Work

- Practical language design and implementation
- Explicit spatial distribution, principals, and authorization (with Jamie Morgenstern)
- Interaction with databases (with João Seco and OutSystems)
- Reasoning about processes (with Jorge Pérez and Henry DeYoung)
  - Observational equivalence via proof theory
  - Towards concurrent type theory
Summary

- A Curry-Howard isomorphism
  - Linear propositions as session types
    - $A \rightarrow B$ (input), $A \otimes B$ (output), $A \& B$ (external choice)
    - $A \oplus B$ (internal choice), $!A$ (replication)
  - Sequent proofs as $\pi$-calculus processes
    with a binary guarded choice and channel forwarding
  - Cut reduction as $\pi$-calculus reduction
- Term-passing extension with a type theory
  - $\forall x: \tau. A$ (term input), $\exists x: \tau. A$ (term output)
- Additional type theory constructs
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