Local vs. Global Illumination

- Local illumination: Phong model (OpenGL)
  - Light to surface to viewer
  - No shadows, interreflections
  - Fast enough for interactive graphics

- Global illumination: Ray tracing
  - Multiple specular reflections and transmissions
  - Only one step of diffuse reflection

- Global illumination: Radiosity
  - All diffuse interreflections; shadows
  - Advanced: combine with specular reflection

Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

Solid Angle

- 2D angle subtended by object O from point x:
  - Length of projection onto unit circle at x
  - Measured in radians (0 to 2π)

- 3D solid angle subtended by O from point x:
  - Area of projection onto unit sphere at x
  - Measured in steradians (0 to 4π)
Radiant Power and Radiosity

- Radiant power $P$
  - Rate at which light energy is transmitted
  - Dimension: $\text{power} = \text{energy} / \text{time}$
- Flux density $\Phi$
  - Radiant power per unit area of the surface
  - Dimension: $\text{power} / \text{area}$
- Irradiance $E$: incident flux density of surface
- Radiosity $B$: exitant flux density of surface
  - Dimension: $\text{power} / \text{area}$
- Flux density at a point $\Phi(x) = \frac{dP}{dx}$

Power at Point in a Direction

- Radiant intensity $I$
  - Power radiated per unit solid angle by point source
  - Dimension: $\text{power} / \text{solid angle}$
- Radiant intensity in direction $\omega$
  - $I(\omega) = \frac{dP}{d\omega}$
- Radiance $L(x, \omega)$
  - Flux density at point $x$ in direction $\omega$
  - Dimension: $\text{power} / (\text{area} \times \text{solid angle})$

Radiance

- Measured across surface in direction $\omega$

$$L(x, \omega) = \frac{d^2P}{d\omega \, dx} = \frac{d^2P}{d\omega \, \cos \theta \, dx}$$

Radiosity and Radiance

- Radiosity $B(x) = \frac{dP}{dx}$
- Radiance $L(x, \omega) = \frac{d^2P}{d\omega \, \cos \theta \, dx}$
- Let $\Omega$ be set of all directions above $x$

$$\overline{B}(x) = \int_{\Omega} L(x, \omega) \cos \theta \, d\omega$$

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Balance of Energy

- Lambertian surfaces (ideal diffuse reflector)
- Divided into $n$ elements
- Variables
  - $A_i$: Area of element $i$ (computable)
  - $B_i$: Radiosity of element $i$ (unknown)
  - $E_i$: Radiant emitted flux density of element $i$ (given)
  - $\rho_i$: Reflectance of element $i$ (given)
  - $F_{ij}$: Form factor from $j$ to $i$ (computable)

$$A_i B_i = A_j E_j + \rho_i \sum_{j=1}^{n} F_{ji} A_j B_j$$
Form Factors

- Form factor $F_{ij}$: Fraction of light leaving element $i$ arriving at element $j$
- Depends on
  - Shape of patches $i$ and $j$
  - Relative orientation of both patches
  - Distance between patches
  - Occlusion by other patches

Form Factor Equation

- Polar angles $\theta$ and $\theta'$ between normals and ray between $x$ and $y$
- Visibility function $v(x,y) = 0$ if ray from $x$ to $y$ is occluded, $v(x,y) = 1$ otherwise
- Distance $r$ between $x$ and $y$

\[ A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos \theta \cos \theta'}{\pi r^2} v(x,y) \, dy \, dx \]

Reciprocity

- Symmetry of form factor

\[ A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos \theta \cos \theta'}{\pi r^2} v(x,y) \, dy \, dx = A_j F_{ji} \]

- Divide earlier radiosity equation

\[ A_i B_i = A_i E_i + \rho_i \sum_{j=1}^{n} F_{ij} A_i B_j \]

by $A_i$

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} (F_{ij} A_j / A_i) B_j \]

Radiosity as a Linear System

- Restate radiosity equation $B_i - \rho_i \sum_{j} F_{ij} B_j = E_i$
- In matrix form

\[
\begin{bmatrix}
1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\
-\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= \begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
\]

- Known: reflectances $\rho_i$, form factors $F_{ij}$, emissions $E_i$
- Unknown: Radiosities $B_i$
- $n$ linear equations in $n$ unknowns

Radiosity "Pipeline"

Visualization

- Radiosity solution is viewer independent
- Can exploit graphics hardware to obtain image
- Convert color on patch to vertex color
- Easy part of radiosity method
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Computing Form Factors

- Visibility critical
- Two principal methods
  - Hemicube: exploit z-buffer hardware
  - Ray casting (can be slow)
- Both exhibit aliasing effects
- For inter-visible elements
  - Many special cases can be solved analytically
  - Avoid full numeric approximation of double integral

Hemicube Algorithm

- Render model onto a hemicube as seen from the center of a patch
- Store patch identifiers j instead of color
- Use z-buffer to resolve visibility
- Efficiently implementable in hardware
- Examples of antialiasing [Chandran et al.]

Wireframe

Classical, No Intensity Interpolation

Antialiasing, No Intensity Interpolation
Radiosity Equation Revisited

- Direct form
  \[ B_i = E_i + \rho_i \sum_j F_{ij} B_j \]

- As matrix equation
  \[
  \begin{bmatrix}
  1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\
  -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn}
  \end{bmatrix}
  \begin{bmatrix}
  B_1 \\
  B_2 \\
  \vdots \\
  B_n
  \end{bmatrix}
  =
  \begin{bmatrix}
  E_1 \\
  E_2 \\
  \vdots \\
  E_n
  \end{bmatrix}
  \]

- Unknown: radiosity \( B_i \)
- Known: emission \( E_i \), form factor \( F_{ij} \), reflect. \( \rho_i \)

Classical Radiosity Algorithms

- Matrix Radiosity
  - Diagonally dominant matrix
  - Use Gauss-Seidel iterative solution
  - Time and space complexity is \( O(n^2) \) for \( n \) elements
  - Memory cost excessive

- Progressive Refinement Radiosity
  - Solve equations incrementally with form factors
  - Time complexity is \( O(n \cdot s) \) for \( s \) iterations
  - Used more commonly (space complexity \( O(n) \))

Matrix Radiosity

- Compute all form factors \( F_{ij} \)
- Make initial approximation to radiosity
  - Emitting elements \( B_i = E_i \)
  - Other elements \( B_i = 0 \)
- Apply equation to get next approximation
  \[ B_i' = E_i + \rho_i \sum_j F_{ij} B_j \]
- Iterate with new approximation
- Intuitively
  - Gather incoming light for each element \( i \)
  - Base new estimate on previous estimate

Radiosity Summary

- Assumptions
  - Opaque Lambertian surfaces (ideal diffuse)
  - Radiosity constant across each element

- Radiosity computation structure
  - Break scene into patches
  - Compute form factors between patches
    - Lighting independent
  - Solve linear radiosity equation
    - Viewer independent
  - Render using standard hardware
Lecture Summary

• Measures of Illumination
• The Radiosity Equation
• Form Factors
• Radiosity Algorithms

Preview

• Next Lecture
  – Radiosity refinements
  – Combining ray tracing and radiosity
• Assignment 7 (Ray Tracer) due April 24
• Different from OpenGL programming (150 pts)