15-462 Computer Graphics I
Lecture 20

Visualization

- Height Fields and Contours
- Scalar Fields
- Volume Rendering
- Vector Fields

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http://www.cs.cmu.edu/~fp/courses/graphics/

Scientific Visualization

- Generally do not start with a 3D model
- Must deal with very large data sets
  - MRI, e.g. $512 \times 512 \times 200 \approx 50$MB points
  - Visible Human $512 \times 512 \times 1734 \approx 433$ MB points
- Visualize both real-world and simulation data
- User interaction
- Automatic search
Types of Data

- Scalar fields (3D volume of scalars)
  - E.g., x-ray densities (MRI, CT scan)
- Vector fields (3D volume of vectors)
  - E.g., velocities in a wind tunnel
- Tensor fields (3D volume of tensors [matrices])
  - E.g., stresses in a mechanical part [Angel 12.7]
- Static or through time

Height Field

- Visualizing an explicit function
  \[ z = f(x,y) \]
- Adding contour curves
  \[ g(x,y) = c \]
Meshes

• Function is sampled (given) at \( x_i, y_i, 0 \leq i, j \leq n \)
• Assume equally spaced
  \[
  x_i = x_0 + i\Delta x \\
  y_j = y_0 + j\Delta y \\
  z_{i,j} = f(x_i, y_j)
  \]
• Generate quadrilateral or triangular mesh
• [Asst 1]

Contour Curves

• Recall: implicit curve \( f(x,y) = 0 \)
• \( f(x,y) < 0 \) inside, \( f(x,y) > 0 \) outside
• Here: contour curve at \( f(x,y) = c \)
• Sample at regular intervals for \( x,y \)
  \[
  x_i = x_0 + i\Delta x \\
  y_j = y_0 + j\Delta y
  \]
• How can we draw the curve?
Marching Squares

- Sample function $f$ at every grid point $x_i, y_j$
- For every point $f_{ij} = f(x_i, y_j)$ either $f_{ij} \leq c$ or $f_{ij} > c$
- Distinguish those cases for each corner $x$
  - White: $f_{ij} \leq c$
  - Black: $f_{ij} > c$
- Now consider cases for curve
- Assume “smooth”
- Ignore $f_{ij} = 0$

Interpolating Intersections

- Approximate intersection
  - Midpoint between $x_i, x_{i+1}$ and $y_j, y_{j+1}$
  - Better: interpolate
- If $f_{ij} = a$ is closer to $c$ than $b = f_{i+1j}$ then intersection is closer to $(x_i, y_j)$:
  $$\frac{x - x_i}{x_{i+1} - x} = \frac{c - a}{b - c}$$
- Analogous calculation for $y$ direction

\[ f_{ij} = a < c \quad c < b = f_{i+1j} \]
Cases for Vertex Labels

16 cases for vertex labels

4 unique mod. symmetries

Ambiguities of Labelings

Ambiguous labels

Different resulting contours

Resolution by subdivision (where possible)
**Marching Squares Examples**

- Ovals of Cassini, $50 \times 50$ grid
  
  \[ f(x, y) = (x^2 + y^2 + a^2)^2 - 4a^2x^2 - b^4 \]
  
  \(a = 0.49, b = 0.5\)

![Midpoint and Interpolation examples](image)

Contour plot of Honolulu data

**Outline**

- Height Fields and Contours
- Scalar Fields
- Volume Rendering
- Vector Fields
Scalar Fields

- Volumetric data sets
- Example: tissue density
- Assume again regularly sampled

\[
\begin{align*}
x_i &= x_0 + i \Delta x \\
y_j &= y_0 + j \Delta y \\
z_k &= z_0 + k \Delta z
\end{align*}
\]

- Represent as voxels

Isosurfaces

- \(f(x,y,z)\) represents volumetric data set
- Two rendering methods
  - Isosurface rendering
  - Direct volume rendering (use all values [next])
- Isosurface given by \(f(x,y,z) = c\)
- Recall implicit surface \(g(x, y, z)\):
  - \(g(x, y, z) < 0\) inside
  - \(g(x, y, z) = 0\) surface
  - \(g(x, y, z) > 0\) outside
- Generalize right-hand side from 0 to c
Marching Cubes

- Display technique for isosurfaces
- 3D version of marching squares
- 14 cube labelings (after elimination symmetries)

Marching Cube Tessellations

- Generalize marching squares, just more cases
- Interpolate as in 2D
- Ambiguities similar to 2D
Volume Rendering

- Sometimes isosurfaces are unnatural
- Use all voxels and transparency ($\alpha$-values)

Ray-traced isosurface

Volume rendering

Surface vs. Volume Rendering

- 3D model of surfaces
- Convert to triangles
- Draw primitives
- Lose or disguise data
- Good for opaque objects

- Scalar field in 3D
- Convert to RGBA values
- Render volume “directly”
- See data as given
- Good for complex objects
Sample Applications

- Medical
  - Computed Tomography (CT)
  - Magnetic Resonance Imaging (MRI)
  - Ultrasound
- Engineering and Science
  - Computational Fluid Dynamic (CFD)
  - Aerodynamic simulations
  - Meteorology
  - Astrophysics

Volume Rendering Pipeline

- Transfer function: from data set to colors and opacities
  - Example: $256 \times 256 \times 64 \times 2 = 4$ MB
  - Example: use colormap (8 bit color, 8 bit opacity)
Transfer Functions

- Transform scalar data values to RGBA values
- Apply to every voxel in volume
- Highly application dependent
- Start from data histogram
- Opacity for emphasis

Transfer Function Example

Mantle Convection

Scientific Computing and Imaging (SCI)
University of Utah
Transfer Function Example

Volume Ray Casting

• Three volume rendering techniques
  – Volume ray casting
  – Splatting
  – 3D texture mapping

• Ray Casting
  – Integrate color through volume
  – Consider lighting (surfaces?)
  – Use regular x,y,z data grid when possible
  – Finite elements when necessary (e.g., ultrasound)
  – 3D-rasterize geometrical primitives
Accumulating Opacity

- $\alpha = 1.0$ is opaque
- Compositing multiple layers according to opacity
- Use local gradient of opacity to detect surfaces for lighting

\[
C_{(i)_{\text{out}}} = C_{(i)_{\text{in}}} \times (1 - \alpha_{(i)}) + C_{(i)} \times \alpha_{(i)}
\]

Trilinear Interpolation

- Interpolate to compute RGBA away from grid
- Nearest neighbor yields blocky images
- Use trilinear interpolation
- 3D generalization of bilinear interpolation
Splattering

- Alternative to ray tracing
- Assign shape to each voxel (e.g., Gaussian)
- Project onto image plane (splat)
- Draw voxels back-to-front
- Composite ($\alpha$-blend)

3D Textures

- Alternative to ray tracing, splatting
- Build a 3D texture (including opacity)
- Draw a stack of polygons, back-to-front
- Efficient if supported in graphics hardware
- Few polygons, much texture memory
Example: 3D Textures
Other Techniques

• Use CSG for cut-away

Acceleration of Volume Rendering

• Basic problem: Huge data sets
• Program for locality (cache)
• Divide into multiple blocks if necessary
  – Example: marching cubes
• Use error measures to stop iteration
• Exploit parallelism
Outline

• Height Fields and Contours
• Scalar Fields
• Volume Rendering
• Vector Fields

Vector Fields

• Visualize vector at each (x,y,z) point
  – Example: velocity field
  – Example: hair
• Hedgehogs
  – Use 3D directed line segments (sample field)
  – Orientation and magnitude determined by vector
• Animation
  – Use for still image
  – Particle systems

Blood flow in human carotid artery
Using Glyphs and Streaks

Glyphs for air flow

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More Flow Examples

Banks and Interrante

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Example: Jet Shockwave

http://www.sci.utah.edu/

Summary

• Height Fields and Contours
• Scalar Fields
  – Isosurfaces
  – Marching cubes
• Volume Rendering
  – Volume ray tracing
  – Splatting
  – 3D Textures
• Vector Fields
  – Hedgehogs
  – Animated and interactive visualization
Preview

• Thursday
  – Non-photo-realistic rendering (NPR)
  – 4:00-5:00 Distinguished Lecture
    Ed Catmull, Pixar, WeH 7500
• Assignment 7 (Ray Tracing) due Thu 4/24
• Assignment 8 (written) out Thu (early!)
• Note: no late hand-in on assignment 8!