

Abstract Syntax

Types	$\tau ::= \tau_1 \multimap \tau_2 \mid \&_{i \in I}(i : \tau_i) \mid \tau_1 \otimes \tau_2 \mid 1 \mid \oplus_{i \in I}(i : \tau_i) \mid \rho\alpha.\tau$	
Contexts	$\Delta ::= \cdot \mid \Delta, x : \tau$	(all variables distinct)
Processes	$P ::= x \leftarrow P ; Q$ $\quad \mid x^w \leftarrow y^R$ $\quad \mid x^W.V \mid \text{case } x^R K$ $\quad \mid x^R.V \mid \text{case } x^W K$	allocate/spawn move $(1, \otimes, \oplus, \rho)$ $(\multimap, \&)$
Small values	$V ::= \langle \rangle \mid \langle a_1, a_2 \rangle \mid i \cdot a \mid \text{fold } a$	
Continuations	$K ::= (\langle \rangle \Rightarrow P) \mid (\langle x_1, x_2 \rangle \Rightarrow P) \mid (i \cdot x_i \Rightarrow P_i)_{i \in I} \mid (\text{fold } x \Rightarrow P)$	
Cell contents	$W ::= V \mid K$	
Configurations	$\mathcal{C} ::= \cdot \mid \mathcal{C}_1, \mathcal{C}_2 \mid \text{proc } d P \mid \text{cell } c W$	

Judgments

$\Delta \Vdash P :: (z : \sigma)$	process P reads from Δ and writes to $z : \sigma$
$\Delta \Vdash \mathcal{C} :: \Delta'$	configuration $\mathcal{C}$ reads from Δ and writes to Δ'
$\mathcal{C} \text{ final}$	configuration $\mathcal{C}$ is final (consists only of cells)

Theorems

Preservation. If $\Delta \Vdash \mathcal{C} :: \Delta'$ and $\mathcal{C} \mapsto \mathcal{C}'$ then $\Delta \Vdash \mathcal{C}' :: \Delta'$.

Progress. If $\cdot \Vdash \mathcal{C} :: \Delta$ then either $\mathcal{C} \mapsto \mathcal{C}'$ for some $\mathcal{C}'$ or $\mathcal{C} \text{ final}$.

Statics, Allocate and Move

$$\frac{\Delta \Vdash P :: (x : \tau) \quad \Delta', x : \tau \Vdash Q :: (z : \sigma)}{\Delta, \Delta' \Vdash (x \leftarrow P ; Q) :: (z : \sigma)} \text{tp/alloc} \qquad \frac{}{y : \tau \Vdash x^W \leftarrow y^R :: (x : \tau)} \text{tp/move}$$

Statics, Positive Types

$\frac{}{\cdot \Vdash x^W.\langle \rangle :: (x : 1)} \text{w/unit}$	$\frac{\Delta \Vdash P :: (z : \sigma)}{\Delta, x : 1 \Vdash \text{case } x^R (\langle \rangle \Rightarrow P) :: (z : \sigma)} \text{r/unit}$
$\frac{}{y : \tau, z : \sigma \Vdash x^W.\langle y, z \rangle :: (x : \tau \otimes \sigma)} \text{w/pair}$	$\frac{\Delta, x_1 : \tau_1, x_2 : \tau_2 \Vdash P :: (z : \sigma)}{\Delta, x : \tau_1 \otimes \tau_2 \Vdash \text{case } x^R (\langle x_1, x_2 \rangle \Rightarrow P) :: (z : \sigma)} \text{r/pair}$
$\frac{(j \in I)}{y : \tau_j \Vdash x^W.(j \cdot y) :: (x : \oplus_{i \in I}(i : \tau_i))} \text{w/tag}$	$\frac{\Delta, y_i : \tau_i \Vdash P_i :: (z : \sigma) \quad (\text{for all } i \in I)}{\Delta, x : \oplus_{i \in I}(i : \tau_i) \Vdash \text{case } x^R (i \cdot y_i \Rightarrow P_i)_{i \in I} :: (z : \sigma)} \text{r/tag}$
$\frac{}{y : [\rho\alpha. \tau/\alpha]\tau \Vdash x^W.(\text{fold } y) :: (x : \rho\alpha. \tau)} \text{w/fold}$	$\frac{\Delta, y : [\rho\alpha. \tau/\alpha]\tau \Vdash P :: (z : \sigma)}{\Delta, x : \rho\alpha. \tau \in \Delta \Vdash \text{case } x^R (\text{fold } y \Rightarrow P) :: (z : \sigma)} \text{r/fold}$

Statics, Negative Types

$\frac{\Delta, y : \tau \Vdash P :: (z : \sigma)}{\Delta \Vdash \text{case } x^W (\langle y, z \rangle \Rightarrow P) :: (x : \tau \multimap \sigma)} \text{w/fun}$	$\frac{}{x : \tau \multimap \sigma, y : \tau \Vdash x^R.\langle y, z \rangle :: (z : \sigma)} \text{r/fun}$
$\frac{\Delta \Vdash P_i :: (z_i : \tau_i) \quad (\text{for all } i \in I)}{\Delta \Vdash \text{case } x^W (i \cdot z_i \Rightarrow P_i)_{i \in I} :: (x : \&_{i \in I}(i : \tau_i))} \text{w/record}$	$\frac{(j \in I)}{x : \&_{i \in I}(i : \tau_i) \Vdash x^R.(j \cdot z) :: (z : \tau_j)} \text{r/record}$

Statics, Configurations

$\frac{\Delta \Vdash P :: (c : \tau)}{\Delta', \Delta \Vdash \text{proc } c P :: (\Delta', c : \tau)} \text{tp/proc}$	
$\frac{\Delta \Vdash c^W.V :: (c : \tau)}{\Delta', \Delta \Vdash \text{cell } c V :: (\Delta', c : \tau)} \text{tp/cell/val}$	$\frac{\Delta \Vdash \text{case } c^W K :: (c : \tau)}{\Delta', \Delta \Vdash \text{cell } c K :: (\Delta', c : \tau)} \text{tp/cell/cont}$
$\frac{}{\Delta \Vdash (\cdot) :: \Delta} \text{tp/empty}$	$\frac{\Delta \Vdash \mathcal{C}_1 :: \Delta_1 \quad \Delta_1 \Vdash \mathcal{C}_2 :: \Delta_2}{\Delta \Vdash (\mathcal{C}_1, \mathcal{C}_2) :: \Delta_2} \text{tp/join}$
$\frac{}{(\cdot) \text{ final}} \text{fin/empty}$	$\frac{\mathcal{C} \text{ final}}{(\mathcal{C}, \text{cell } c W) \text{ final}} \text{fin/cell}$

Dynamics

	$\text{proc } d (x \leftarrow P ; Q)$	$\mapsto$	$\text{proc } c ([c/x]P), \text{proc } d ([c/x]Q)$	(alloc/spawn, c fresh)
cell $c W$,	$\text{proc } d (d^W \leftarrow c^R)$	$\mapsto$	cell $d W$	(move)
	$\text{proc } d (d^W.V)$	$\mapsto$	cell $d V$	(write: $\otimes, 1, \oplus, \rho$)
cell $c V$,	$\text{proc } d (\text{case } c^R K)$	$\mapsto$	$\text{proc } d (V \triangleright K)$	(read: $\otimes, 1, \oplus, \rho$)
	$\text{proc } d (\text{case } d^W K)$	$\mapsto$	cell $d K$	(write: $\multimap, \&$)
cell $c K$,	$\text{proc } d (c^R.V)$	$\mapsto$	$\text{proc } d (V \triangleright K)$	(read: $\multimap, \&$)

$$\begin{aligned}
\langle \rangle &\triangleright (\langle \rangle \Rightarrow P) &&= P \\
\langle c_1, c_2 \rangle &\triangleright (\langle x_1, x_2 \rangle \Rightarrow P) &&= [c_1/x_1, c_2/x_2]P \\
j \cdot c &\triangleright (i \cdot x_i \Rightarrow P_i)_{i \in I} &&= [c/x_j]P_j \\
\text{fold } c &\triangleright (\text{fold } x \Rightarrow P) &&= [c/x]P
\end{aligned}$$