

# Constructive Logic (15-317), Spring 2023

## Recitation 2

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### 1 Verifications & Uses

Consider the  $\heartsuit$  connective:<sup>1</sup>

$$\frac{\overline{A \text{ true}}^v \quad \overline{A \text{ true}}^w \quad \vdots \quad \overline{B \text{ true}} \quad \overline{C \text{ true}}}{\heartsuit(A, B, C) \text{ true}} \heartsuit I^{v,w}$$

$$\frac{\heartsuit(A, B, C) \text{ true} \quad \overline{B \text{ true}}^u \quad \vdots \quad A \text{ true} \quad D \text{ true}}{D \text{ true}} \heartsuit E1^u \quad \frac{\heartsuit(A, B, C) \text{ true} \quad \overline{C \text{ true}}^u \quad \vdots \quad A \text{ true} \quad D \text{ true}}{D \text{ true}} \heartsuit E2^u$$

**Task 1.** Give appropriate rules for  $\heartsuit$  in verifications & uses.

**Solution:**

$$\frac{\overline{A \downarrow}^v \quad \overline{A \downarrow}^w \quad \vdots \quad B \uparrow \quad C \uparrow}{\heartsuit(A, B, C) \uparrow} \heartsuit I^{v,w}$$

$$\frac{\heartsuit(A, B, C) \downarrow \quad \overline{B \downarrow}^u \quad \vdots \quad A \uparrow \quad D \uparrow}{D \uparrow} \heartsuit E1^u \quad \frac{\heartsuit(A, B, C) \downarrow \quad \overline{C \downarrow}^u \quad \vdots \quad A \uparrow \quad D \uparrow}{D \uparrow} \heartsuit E2^u$$

<sup>1</sup>in Latex: `\heartsuit`

Looking at the intros rules, as with all other intro rules, the conclusion should be a verification. Similarly, both B and C need verifications, so we continue upward until we reach our assumptions v and w which we "use" (as with any hypothetical judgements).

Now looking at the Elim rules, they are similar to the elim rules for implication. The term we are eliminating needs to be a use, but we are still looking for a derivation of A and a derivation of D, so both of those need to be verifications. Moving upwards, somewhere in the middle there will be a switch, and we will reach our assumptions which are uses.

**Task 2.** Using Dcheck, give a derivation of the judgement:

$$(A \supset B \supset C) \supset ((A \wedge B) \supset C) \uparrow$$

**Solution:** Separate DCheck File

## 2 Proof Terms/Proofs as Programs

**Task 3.** Consider the judgement in task 2. What would be the appropriate type signature for a program in SML? Write a program in SML that corresponds to that type signature. **Solution:**

```
- fun fst (a,b) = a;
val fst = fn : 'a * 'b -> 'a
- fun snd (a,b) = b;
val snd = fn : 'a * 'b -> 'b
```

```
- fun uncurry f x = f (fst x) (snd x);
val uncurry = fn : ('a -> 'b -> 'c) -> 'a * 'b -> 'c
```

First two functions are just defining fst and snd since they are not part of the SML library.

**Task 4.** Consider again the  $\heartsuit$  connective from the previous section. Let us use the syntax  $\text{hearti}(x.M, y.N)$  for the proof term for  $\heartsuit I$ , and the syntax  $\text{heartel}(M, N, x.P)$  and  $\text{hearter}(M, N, x.P)$  for the proof terms for  $\heartsuit E1$  and  $\heartsuit E2$ .

Give proof term deduction rules corresponding to  $\heartsuit I$ ,  $\heartsuit E1$ , and  $\heartsuit E2$ .

**Solution:**

$$\frac{\frac{\overline{x : A} \quad \overline{y : A}}{\vdots \quad \vdots} \quad \frac{M : B \quad N : C}{\text{hearti}(x.M, y.N) : \heartsuit(A, B, C)}}{\heartsuit I}$$

$$\frac{M : \heartsuit(A, B, C) \quad N : A \quad \frac{\overline{x : B}}{\vdots} \quad P : D}{\text{heartel}(M, N, x.P) : D} \heartsuit E1$$

$$\frac{M : \heartsuit(A, B, C) \quad N : A \quad \overline{x : C} \quad \vdots \quad P : D}{\text{hearter}(M, N, x.P) : D} \heartsuit E2$$