

Bidirectional Type-Checking

Judgments.

$M \Leftarrow A$ M checks against A
 $M \Rightarrow A$ M synthesizes A

Metatheorems.

1. For all M and A , either $M \Leftarrow A$ or not.
2. For all M , either there is an A with $M \Rightarrow A$ or not.

Rules.

$$\frac{M \Rightarrow P' \quad P' = P}{M \Leftarrow P} \downarrow \uparrow^*$$

$$\frac{M \Leftarrow A \quad N \Leftarrow B}{\langle M, N \rangle \Leftarrow A \wedge B} \wedge I \quad \frac{M \Rightarrow A \wedge B}{\mathbf{fst} M \Rightarrow A} \wedge E_1 \quad \frac{M \Rightarrow A \wedge B}{\mathbf{snd} M \Rightarrow B} \wedge E_2$$

$$\frac{\begin{array}{c} \overline{x} \quad x \\ x \Rightarrow A \\ \vdots \\ M \Leftarrow B \end{array}}{\lambda x. M \Leftarrow A \supset B} \supset I^x \quad \frac{M \Rightarrow A \supset B \quad N \Leftarrow A}{M N \Rightarrow B} \supset E$$

$$\frac{M \Leftarrow A}{\mathbf{inl} M \Leftarrow A \vee B} \vee I_1 \quad \frac{M \Leftarrow B}{\mathbf{inr} M \Leftarrow A \vee B} \vee I_2 \quad \frac{\begin{array}{c} \overline{u} \quad u \quad \overline{w} \quad w \\ u \Rightarrow A \quad w \Rightarrow B \\ \vdots \quad \vdots \\ M \Rightarrow A \vee B \quad N \Leftarrow C \quad P \Leftarrow C \end{array}}{\mathbf{case}(M, u. N, w. P) \Leftarrow C} \vee E^{u,w}$$

$$\frac{}{\langle \rangle \Leftarrow \top} \top I \quad (\text{no } \top E)$$

$$(\text{no } \perp I) \quad \frac{M \Rightarrow \perp}{\mathbf{abort} M \Leftarrow C} \perp E$$