# Constructive Logic (15-317), Fall 2017
# Assignment 9: Substructural Logic

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Due Tuesday, November 21, 2017

This assignment is due at the beginning of class on the above date and must be submitted electronically via Autolab. Submit your homework as a **PDF** file containing your written solutions. *Do not* submit a tar file.

**No points can be awarded for submissions that are not PDF files. After submitting via Autolab, please check your submission's contents to make sure they are what you expect!**

## 1 Linear Reasoning

As a typesetting convention in this assignment, we represent ephemeral judgments as *abc* and persistent judgments as $\underline{\textit{abc}}$.

The solution to each task in this section should have three explicit parts:

1. a characterisation of the initial state,
2. rules defining the condition of the problem,
3. a (simple) characterisation and an interpretation of the final state.

You are free to design your initial state to contain linear or persistent propositions that are easily derived from the given persistent $\underline{\textit{edge}}(x, y)$ and $\underline{\textit{node}}(v)$ predicates.

### 1.1 Graph Theory

A graph $G$ consists of a collection of edges $\underline{\textit{edge}}(u, v)$ and a collection of vertices $\underline{\textit{node}}(v)$ such that if $\underline{\textit{edge}}(x, y)$ then $\underline{\textit{node}}(x)$ and $\underline{\textit{node}}(y)$. You may assume throughout that $G$ is *undirected*, that is to say, that $\underline{\textit{edge}}(x, y)$ whenever $\underline{\textit{edge}}(y, x)$. We say that $G$ is *connected* if for all $\underline{\textit{node}}(x)$ and $\underline{\textit{node}}(y)$, $x = y$ or there exists a sequence of edges $\underline{\textit{edge}}(x, x_1), \underline{\textit{edge}}(x_1, x_2), \ldots, \underline{\textit{edge}}(x_{n-1}, x_n), \underline{\textit{edge}}(x_n, y)$ forming a path between $x$ and $y$.

A *spanning tree* of a graph $G$ is a tree that is a subgraph of $G$ and contains all of the vertices of $G$. Equivalently, it is a connected subgraph of $G$ containing all of the vertices of $G$ but no cycle.

**Task 1** (5 points)**.** Give rules that will find a spanning tree of a connected graph. Make sure your solution includes the three explicit parts described at the beginning of this section.

A graph is *bipartite* if its vertices can be partitioned into two sets $L$ and $R$ such that no two vertices in the same set have an edge between them.

**Task 2** (10 points)**.** Give rules that will decide if a connected graph is bipartite. Make sure your solution includes the three explicit parts described at the beginning of this section.

A *matching* for a graph is a subset $M$ of the edges such that no two edges in $M$ have a common vertex.

**Task 3** (5 points)**.** Give rules that will find a non-trivial matching of an arbitrary graph with at least one edge. Make sure your solution includes the three explicit parts described at the beginning of this section.

## 2 Ordered Reasoning

In class, we studied the Lambek calculus as a logical means for parsing natural language. We observed that the *over* and *under* connectives were akin to implication, and that the *fuse* connective was akin to conjunction. We saw that these connectives had analogues of currying, namely, that we had the following bi-entailments:

$$y \backslash (x \backslash z) \dashv\vdash (x \bullet y) \backslash z$$
$$(z / x) / y \dashv\vdash z / (y \bullet x).$$

**Task 4** (20 points)**.** Consider a connective $x \circ y$ (pronounced $x$ *twist* $y$) defined in the original style of Lambek by

$$\frac{x \circ y}{y \quad x}\ \mathsf{twist}$$

Our goal is to study this connective and find its analogues of currying.

1. Define the right and left rules for $x \circ y$.
2. Prove or refute that $x \circ y \vdash y \bullet x$.
3. Find a curried equivalent $A(x, y, z)$ of $(x \circ y) \backslash z$ and prove $A(x, y, z) \vdash (x \circ y) \backslash z$ and $(x \circ y) \backslash z \vdash A(x, y, z)$.