Constructive Logic (15-317), Fall 2017
Assignment 4: Sequent Calculus

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Due Tuesday, October 10, 2017

This assignment is due at the beginning of class on the above date and must be submitted electronically via Autolab.

1 Sequent Proofs

Task 1 (5 points)**.** Provide a proof of the following proposition in the sequent calculus. You may silently drop antecedents you no longer need in the remainder of the sequent proof, but beware of dropping antecedents too early!

$$\Longrightarrow (A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$$

2 Negation

Task 2 (2 points)**.** Show that the following two rules for negation are *derived rules* in the sequent calculus under the usual definition of $\neg A \triangleq A \supset \bot$.

$$\frac{\Gamma, A \Longrightarrow \bot}{\Gamma \Longrightarrow \neg A}\ \neg R \qquad\qquad \frac{\Gamma, \neg A \Longrightarrow A}{\Gamma, \neg A \Longrightarrow C}\ \neg L$$

Task 3 (10 points)**.** The rules for negation in Task 1 are not only derived rules, but they are also sufficient if we treat $\neg A$ as a primitive rather than as defined. For each of the following propositions, supply a proof in the sequent calculus using $\neg R$ and $\neg L$ or indicate that no proof exists. Again, you may silently drop antecedents you no longer need in the rest of the proof, but beware of dropping antecedents too early!

a. $\Longrightarrow \neg\neg(A \vee \neg A)$

b. $\Longrightarrow \neg\neg((\neg\neg A) \supset A)$

3 Admissibility of Cut

Task 4 (5 points)**.** Extend the proof of the admissibility of cut from Lecture 10 by providing the following case in the same detailed style:

Case: $\mathcal{D}$ ends in $\vee R_2$ and $\mathcal{E}$ ends in $\vee L$, where $\vee L$ is applied on the principal formula of the cut.

The solution template contains the case from the lecture notes where $\mathcal{D}$ ends in $\wedge R$ and $\mathcal{E}$ ands in $\wedge L_1$ on the principal formula, as an aid in typesetting.

4 Applications of Cut Admissibility

Task 5 (5 points)**.** Using the admissibility of cut, prove:

If $\Gamma, A \wedge B \Longrightarrow C$ *then* $\Gamma, A, B \Longrightarrow C$.

In particular, you should not use any induction in your argument.