

# Constructive Logic (15-317), Fall 2017

## Assignment 1: Natural Deduction & Constructivity

Course Staff

Due Tuesday, September 12, 2017

This assignment is due at the beginning of class on the above date and it must be submitted electronically at autolab. Submit your homework as a tar archive containing the following files:

- hw1.pdf (your written solutions);
- hw1.1a.tut, ..., hw1.1e.tut (your Tutch solutions for task 1); and
- hw1.3a.tut and hw1.3b.tut (your Tutch solutions for task 3).

### 1 Tutch Proofs

**Task 1** (10 points). Prove the following theorems using Tutch. Place the proof for part a in hw1.1a.tut, part b in hw1.1b.tut, ..., and part e in hw1.1e.tut.

- proof absurdity :  $A \ \& \ \sim A \Rightarrow B$ ;
- proof sCombinator :  $(A \Rightarrow B) \Rightarrow (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow C)$ ;
- proof deMorgin :  $\sim(A \ | \ B) \Rightarrow \sim A \ \& \ \sim B$ ;
- proof deMorgout :  $\sim A \ \& \ \sim B \Rightarrow \sim(A \ | \ B)$ ;
- proof covariance :  $(A \Rightarrow B) \Rightarrow (X \Rightarrow (Y \ | \ (A \ \& \ Z))) \Rightarrow (X \Rightarrow (Y \ | \ (B \ \& \ Z)))$ ;

Recall that in Tutch, the constant  $F$  means  $\perp$  and the notation  $\sim A$  is a shorthand for  $A \Rightarrow F$ , in the same way as  $\neg A$  is a notation for  $A \supset \perp$ ;  $A \ | \ B$  is the notation for  $A \vee B$ .

We have provided you with requirements files to check your progress against. For example, you can check your progress for part a by running

```
$ tutch -r ./hw1.1a.req hw1.1a.tut
```

## 2 The Wheat and the Chaff

**Task 2** (10 points). The skill of detecting bogus arguments is critical in mathematics. The fallacy of *denying the antecedent* occurs occasionally in everyday bogus arguments. It looks like this:

$$(A \supset B) \supset (\neg A \supset \neg B) \text{true} \quad (*)$$

Show that this is bogus in the case where  $\neg A \wedge B \text{true}$  by proving:

$$(\neg A \wedge B) \supset ((A \supset B) \supset (\neg A \supset \neg B)) \supset \perp \text{true}$$

Once again, recall that  $\neg B$  is shorthand for  $B \supset \perp$ . Be sure to label each inference rule in your proof.

## 3 Constructive and Classical Reasoning

By default, proofs in Tutch must be intuitionistic. However, it is possible to use Tutch to check a classical proof by using the `classical proof` declaration form; this form adds the facility to reason by contradiction.

Proof by contradiction is when you prove  $A$  by assuming  $\neg A$  and deriving a contradiction. The paradigmatic example of proof by contradiction is captured in the following Tutch code:

```
classical proof byContradiction :  $\sim\sim A \Rightarrow A =$ 
begin
  [  $\sim\sim A$ ;
    [  $\sim A$ ;
      F];
    A
  ];
 $\sim\sim A \Rightarrow A$ 
end;
```

Tip: do not confuse *proof by contradiction* with *reductio ad absurdum*; the latter refers to concluding  $\neg A$  from  $A \supset \perp$ , and is completely constructive.

**Task 3** (20 points). Which directions of the following equivalence can you prove using the rules of intuitionistic/constructive logic? If a constructive proof is not possible, is there a classical proof?

$$(A \supset B) \supset C \Leftrightarrow (A \vee C) \wedge (B \supset C) \text{true}$$

To answer this question, try to prove the following theorems in Tutch. Place the proof for part a in `hw1_3a.tut` and part b in `hw1_3b.tut`.

a. `proof right` :  $((A \Rightarrow B) \Rightarrow C) \Rightarrow (A \mid C) \ \& \ (B \Rightarrow C)$

b. `proof left` :  $((A \mid C) \ \& \ (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow C)$

If the proof cannot be carried out, replace the proof declaration with `classical proof` and try again. Full points will not be awarded for a classical proof when a constructive one is possible.

We have provided you with requirements files to check your progress against. For example, you can check your progress for part a by running

```
$ tutch -r ./hw1_3a.req hw1_3a.tut
```

Please note that until the submission deadline, Autolab will only check for the existence of valid Tutch proofs and will assign the same number of points to both constructive and classical proofs. We will adjust the points awarded for classical versus constructive proofs after the submission deadline.