

# 15-317: Constructive Logic

## Recitation 1

The following logic puzzle is derived from the game Clue<sup>TM</sup>. A player has various pieces of information regarding the perpetrator, the weapon, and the location of a murder:

*If Mr. Green did it, then it was with the candle-stick in the billiard room or with the revolver in the hall. If it wasn't in the billiard room, then Miss Scarlet did it. Either it was done with the revolver, or in the hall.*

From these facts, a player must make inferences about who committed the crime, such as:

*Therefore Mr. Green didn't do it.*

**Task 1** Write out the statements above in the language of propositional logic, using the following notation:

$G$	=	Mr. Green did it
$S$	=	Miss Scarlet did it
$R$	=	the weapon was a revolver
$C$	=	the weapon was a candle-stick
$B$	=	it happened in the billiard room
$H$	=	it happened in the hall
$P \wedge Q$	=	$P$ and $Q$
$P \vee Q$	=	$P$ or $Q$
$P \supset Q$	=	$P$ implies $Q$
$\neg P$	=	not $P$

### Solution

- (a) *If Mr. Green did it, then it was with the candle-stick in the billiard room or with the revolver in the hall.*  
 $G \supset (C \wedge B) \vee (R \wedge H).$
- (b) *If it wasn't in the billiard room, then Miss Scarlet did it.*  
 $\neg B \supset S.$
- (c) *Either it was done with the revolver, or in the hall.*  
 $R \vee H.$
- (d) *Mr. Green didn't do it.*  
 $\neg G.$

Note: (c) interprets “either ... or ...” inclusively—it’s possible that it was both done with the revolver and in the hall—because we write  $\vee$  for “inclusive or”.

Also, it is implicit in the rules of Clue that there is only one murderer, one weapon, and one location:

(e)  $\neg(G \wedge S)$ .

(f)  $\neg(R \wedge C)$ .

(g)  $\neg(B \wedge H)$ .

**Task 2** Is it correct to conclude that Mr. Green didn’t do it? Why or why not?

**Solution** Yes, Mr. Green can be acquitted.

We prove  $\neg G$  by assuming  $G$  and deriving false ( $\perp$ ).

(1) Assume  $G$ .

(2) By (a) and (1), we know that  $(C \wedge B) \vee (R \wedge H)$ .

(3) First we consider the case that  $C \wedge B$ .

(4) From (3), we know  $C$  ...

(5) and  $B$ .

(6) Next, we case-analyze (c).

(7) In the first case, we know  $R$

(8) Then we have  $R \wedge C$  from (7) and (4)

(9) Thus, we conclude  $\perp$  from (8) and (f)

(10) In the next case, we know  $H$ .

(11) Then we have  $B \wedge H$  from (10) and (5).

(12) Thus, we conclude  $\perp$  from (11) and (g)

(13) Next we consider the case that  $R \wedge H$ .

(14) From (13) we know  $H$ .

(15) Next, we prove  $\neg B$ :

(16) Assume  $B$ .

(17) Then from (14) and (16) we have  $B \wedge H$ .

(17) Then from (17) and (g) we conclude  $\perp$ .

(18) By (15) and (b), we conclude  $S$ .

(19) By (18) and (1), we know  $G \wedge S$ .

(20) By (19) and (e), we conclude  $\perp$ .

Tomorrow, we will start learning the formal rules that justify these inferences.