Assignment 3:
Quantifiers

15-317: Constructive Logic

Out: Thursday, September 18, 2008
Due: Thursday, September 25, 2008, before class

1 Tutch Proofs (28 pts)

For each of the following propositions:

1. Determine whether it is true (constructively).
2. If the proposition is true, give a Tutch proof, and (sometimes) a proof term, as indicated below.

```
proof compose : (!x:t.A(x) => B(x)) => (!x:t.B(x) => C(x)) => (!x:t.A(x) => C(x))
term  compose : (!x:t.A(x) => B(x)) => (!x:t.B(x) => C(x)) => (!x:t.A(x) => C(x))

proof dea1 : (?x:t.A(x) & B(x)) => (?x:t.A(x)) & (?x:t.B(x))
proof dea2 : ((?x:t.A(x)) & (?x:t.B(x))) => (?x:t.A(x) & B(x))

proof dao1 : (!x:t.A(x) | B(x)) => (!x:t.A(x)) | (!x:t.B(x))
proof dao2 : ((!x:t.A(x)) | (!x:t.B(x))) => (!x:t.A(x) | B(x))

proof daa1 : (!x:t.A(x) & B(x)) => (!x:t.A(x)) & (!x:t.B(x))
term  daa1 : (!x:t.A(x) & B(x)) => (!x:t.A(x)) & (!x:t.B(x))
proof daa2 : ((!x:t.A(x)) & (!x:t.B(x))) => (!x:t.A(x) & B(x))
term  daa2 : ((!x:t.A(x)) & (!x:t.B(x))) => (!x:t.A(x) & B(x))

proof deo1 : (?x:t.A(x) | B(x)) => (?x:t.A(x)) | (?x:t.B(x))
term  deo1 : (?x:t.A(x) | B(x)) => (?x:t.A(x)) | (?x:t.B(x))
proof deo2 : ((?x:t.A(x)) | (?x:t.B(x))) => (?x:t.A(x) | B(x))
term  deo2 : ((?x:t.A(x)) | (?x:t.B(x))) => (?x:t.A(x) | B(x))

proof dm1 : ~(?x:t.A(x)) => !x:t.~A(x)
term  dm1 : ~(?x:t.A(x)) => !x:t.~A(x)
proof dm2 : (!x:t.~A(x)) => ~(?x:t.A(x))
term  dm2 : (!x:t.~A(x)) => ~(?x:t.A(x))
```

Here is the new Tutch syntax you'll need:

- $\forall x:\tau.A$ is written `!x:t.A`
- $\exists x:\tau.A$ is written `?x:t.A`
- The existential elimination rule requires a hypothetical with two assumptions. These are written by separating the assumptions with a comma. For example:

```
proof exp : (?x:t.A(x)) => ?x:t.A(x) =
begin
[(?x:t.A(x));
 [a:t , A(a);
  ?x:t.A(x)];
 ?x:t.A(x)];
(?x:t.A(x)) => ?x:t.A(x);
end;
```

- Proof terms for the quantifiers are written as follows:

Rule	Proof Term
$\forall I$	`fn x => M`
$\forall E$	`M N`
$\exists I$	`(M , N)`
$\exists E$	`let (x,u) = M in N`

2 Verifications and Uses (12 pts, Written)

For each of the following propositions:

1. Determine whether it is true (constructively).
2. If the proposition is true, give a verification/use derivation.
3. If the proposition is not true, argue that there is no verification/use derivation.

See the Lecture 6 notes for the verification/use rules for the quantifiers. See the Lecture 3 notes for the verification/use rules for the propositional connectives.

```
dea1 : (?x:t.A(x) & B(x)) => (?x:t.A(x)) & (?x:t.B(x))
dea2 : ((?x:t.A(x)) & (?x:t.B(x))) => (?x:t.A(x) & B(x))
dao1 : (!x:t.A(x) | B(x)) => (!x:t.A(x)) | (!x:t.B(x))
dao2 : ((!x:t.A(x)) | (!x:t.B(x))) => (!x:t.A(x) | B(x))
```

3 Handin Instructions

- To run Tutch with the requirements files, run

```
/afs/andrew/course/15/317/bin/tutch -r hw04.req <your file>
```

This uses the requirements file `/afs/andrew/course/15/317/req/hw04.req`.

- To submit your Tutch proofs, run

  ```
  /afs/andrew/course/15/317/bin/submit -r hw04.req <your file>
  ```

  To check the status of your submission, run `/afs/andrew/course/15/317/bin/status hw03`.

  It is expected for Tutch to report that you have unsolved problems corresponding to those propositions in Problem 1 that you think are false.

- Submit your written work at the beginning of class, or, if you wish to do an electronic handin, copy a PDF to

  ```
  /afs/andrew/course/15/317/submit/<yourid>/hw03.pdf
  ```