

Assignment 1: Natural Deduction

15-317: Constructive Logic

Out: Thursday, September 4, 2008
Due: Thursday, September 11, 2008, before class

Welcome to 15-317!

1 Bulletin Board (3 pts)

You are responsible for all announcements posted to the bulletin board. In particular, we may post clarifications and corrections to assignments there. You should check the bboard often.

The course bulletin board is

`academic.cs.15-317`

You will find a “magic number” in a post there.

Task 1 (3 pts). What is that number?

2 Tutch Proofs (25 pts)

Prove the following theorems using Tutch:

```
proof k : A => (B => A)
proof andcomm : (A & B) => B & A
proof curry : ((A & B) => C) => (A => B => C)
proof distribandorl : ((A & B) | C) => (A | C) & (B | C)
proof clue : (G => (C & B) | (R & H))
    => (~B => S)
    => (R | H)
    => ~(G & S)
    => ~(R & C)
    => ~(B & H)
    => (~G)
```

See the Recitation 1 notes for a paper proof of `clue`, and see the Recitation 2 notes for example Tutch proofs. On Andrew machines, you can check that you proved the right theorems by running

```
/afs/andrew/course/15/317/bin/tutch -r hw01.req <your file>
```

This checks your work against the requirements file `/afs/andrew/course/15/317/req/hw01.req`.

3 Local Soundness and Completeness (12 pts)

See the Lecture 3 (Harmony) notes for a discussion of local soundness and completeness.

3.1 Hearts

Consider a connective defined by the following rules:

$$\frac{A \text{ true}}{A \heartsuit B \text{ true}} \heartsuit I_L \quad \frac{B \text{ true}}{A \heartsuit B \text{ true}} \heartsuit I_R \quad \frac{\overline{A \text{ true}}^u, \overline{B \text{ true}}^v, \dots, C \text{ true}}{C \text{ true}} \heartsuit E^{u,v}$$

Task 3 (3 pts). Is this connective locally sound? If so, show the reduction; if not, explain (informally) why no such reduction exists.

Task 4 (3 pts). Is this connective locally complete? If so, show the expansion; if not, explain (informally) why no such expansion exists.

3.2 Clubs

Consider a connective defined by the following rules:

$$\frac{\overline{A \text{ true}}^u, \dots, B \text{ true}}{\clubsuit(A, B, C) \text{ true}} \clubsuit I_L^u \quad \frac{\overline{A \text{ true}}^u, \dots, C \text{ true}}{\clubsuit(A, B, C) \text{ true}} \clubsuit I_R^u \quad \frac{\overline{B \text{ true}}^u, \overline{C \text{ true}}^v, \dots, D \text{ true}}{D \text{ true}} \clubsuit E^{u,v}$$

Task 1 (3 pts). Is this connective locally sound? If so, show the reduction; if not, explain (informally) why no such reduction exists.

Task 2 (3 pts). Is this connective locally complete? If so, show the expansion; if not, explain (informally) why no such expansion exists.

4 Handin Instructions

- To submit your Tutch proofs, run

```
/afs/andrew/course/15/317/bin/submit -r hw01.req <your file>
```

To check the status of your submission, you may run `/afs/andrew/course/15/317/bin/status hw01`.

If you get an error, it is probably because your submission directory does not exist (because you were not registered for the course at the time these directories were made); e-mail Dan (drl@cs.cmu.edu).

- Submit your written work at the beginning of class.