

## Radiosity

Measures of Illumination  
The Radiosity Equation  
Form Factors  
Radiosity Algorithms  
Handout: [Watt & Watt, Ch 11]

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Frank Pfenning  
Carnegie Mellon University  
<http://www.cs.cmu.edu/~fp/courses/graphics/>

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## Local vs. Global Illumination

- Local illumination: Phong model (OpenGL)
  - Light to surface to viewer
  - No shadows, interreflections
  - Fast enough for interactive graphics
- Global illumination: Ray tracing
  - Multiple specular reflections and transmissions
  - Only one step of diffuse reflection
- Global illumination: Radiosity
  - All diffuse interreflections; shadows
  - Advanced: combine with specular reflection

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## Image vs. Object Space

- Image space: Ray tracing
  - Trace backwards from viewer
  - View-dependent calculation
  - Result: rasterized image (pixel by pixel)
- Object space: Radiosity
  - Assume only diffuse-diffuse interactions
  - View-independent calculation
  - Result: 3D model, color for each surface patch
  - Can render with OpenGL

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## Classical Radiosity Method

- Divide surfaces into patches (elements)
- Model light transfer between patches as system of linear equations
- Important assumptions:
  - Reflection and emission are diffuse
    - Recall: diffuse reflection is equal in all directions
    - So radiance is independent of direction
  - No participating media (no fog)
  - No transmission (only opaque surfaces)
  - Radiosity is constant across each element
  - Solve for R, G, B separately

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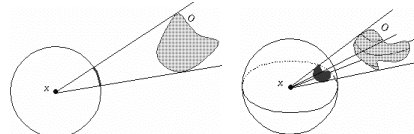
## Outline

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

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## Solid Angle

- 2D angle subtended by object O from point x:
  - Length of projection onto unit circle at x
  - Measured in radians (0 to  $2\pi$ )
- 3D solid angle subtended by O from point x:
  - Area of projection onto unit sphere at x
  - Measured in steradians (0 to  $4\pi$ )



J. Stewart

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## Radiant Power and Radiosity

- Radiant power P
  - Rate at which light energy is transmitted
  - Dimension: power = energy / time
- Flux density  $\Phi$ 
  - Radiant power per unit area of the surface
  - Dimension: power / area
- Irradiance E: incident flux density of surface
- Radiosity B: exitant flux density of surface
  - Dimension: power / area
- Flux density at a point  $\Phi(x) = dP/dx$

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## Power at Point in a Direction

- Radiant intensity I
  - Power radiated per unit solid angle by point source
  - Dimension: power / solid angle
- Radiant intensity in direction  $\omega$ 
  - $I(\omega) = dP/d\omega$
- Radiance  $L(x, \omega)$ 
  - Flux density at point x in direction  $\omega$
  - Dimension: power / (area  $\times$  solid angle)

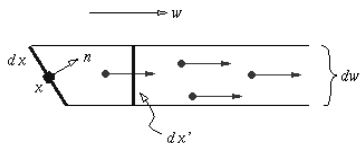
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## Radiance

- Measured across surface in direction  $\omega$



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- For angle  $\theta$  between  $\omega$  and normal  $\mathbf{n}$

$$L(x, \omega) = \frac{d^2 P}{d\omega dx'} = \frac{d^2 P}{d\omega \cos \theta dx}$$

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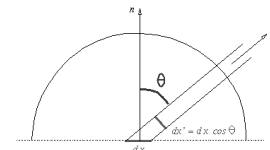
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## Radiosity and Radiance

- Radiosity  $B(x) = dP / dx$
- Radiance  $L(x, \omega) = d^2 P / d\omega \cos \theta dx$
- Let  $\Omega$  be set of all directions above x

$$B(x) = \int_{\Omega} L(x, \omega) \cos \theta d\omega$$



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## Balance of Energy

- Lambertian surfaces (ideal diffuse reflector)
- Divided into n elements
- Variables
  - $A_i$  Area of element i (computable)
  - $B_i$  Radiosity of element i (unknown)
  - $E_i$  Radiant emitted flux density of element i (given)
  - $\rho_i$  Reflectance of element i (given)
  - $F_{ji}$  Form factor from j to i (computable)

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^n F_{ji} A_j B_j$$

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## Form Factors

- Form factor  $F_{ij}$ : Fraction of light leaving element  $i$  arriving at element  $j$
- Depends on
  - Shape of patches  $i$  and  $j$
  - Relative orientation of both patches
  - Distance between patches
  - Occlusion by other patches

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## Form Factor Equation

- Polar angles  $\theta$  and  $\theta'$  between normals and ray between  $x$  and  $y$
- Visibility function  $v(x,y) = 0$  if ray from  $x$  to  $y$  is occluded,  $v(x,y) = 1$  otherwise
- Distance  $r$  between  $x$  and  $y$

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos \theta \cos \theta'}{\pi r^2} v(x,y) dy dx$$

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## Reciprocity

- Symmetry of form factor

$$A_i F_{ij} = \int_{x \in P_i} \int_{y \in P_j} \frac{\cos \theta \cos \theta'}{\pi r^2} v(x,y) dy dx = A_j F_{ji}$$

- Divide earlier radiosity equation

$$A_i B_i = A_i E_i + \rho_i \sum_{j=1}^n F_{ji} A_j B_j$$

$$\text{by } A_i \quad B_i = E_i + \rho_i \sum_j (F_{ji} A_j / A_i) B_j$$

$$= E_i + \rho_i \sum_j F_{ij} B_j$$

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## Radiosity as a Linear System

- Restate radiosity equation  $B_i - \rho_i \sum_j F_{ij} B_j = E_i$
- In matrix form

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

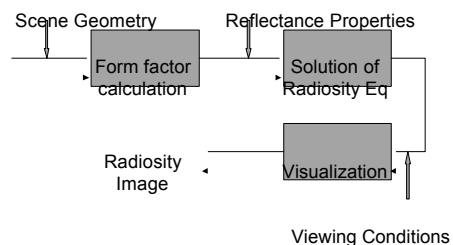
- Known: reflectances  $\rho_i$ , form factors  $F_{ij}$ , emissions  $E_i$
- Unknown: Radiosities  $B_i$
- $n$  linear equations in  $n$  unknowns

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## Radiosity "Pipeline"



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## Visualization

- Radiosity solution is viewer independent
- Can exploit graphics hardware to obtain image
- Convert color on patch to vertex color
- Easy part of radiosity method

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## Computing Form Factors

- Visibility critical
  - Hemicube: exploit z-buffer hardware
  - Ray casting (can be slow)
  - Both exhibit aliasing effects
- For inter-visible elements
  - Many special cases can be solved analytically
  - Avoid full numeric approximation of double integral

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## Hemicube Algorithm

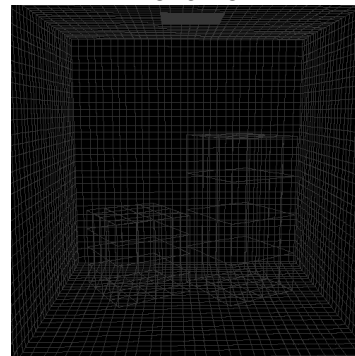
- Render model onto a hemicube as soon from the center of a patch
- Store patch identifiers  $j$  instead of color
- Use z-buffer to resolve visibility
- Efficiently implementable in hardware
- Examples of antialiasing [Chandran et al.]

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## Wireframe

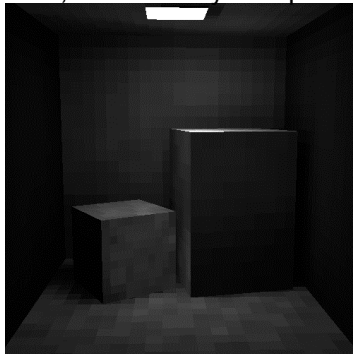


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## Classical, No Intensity Interpolation

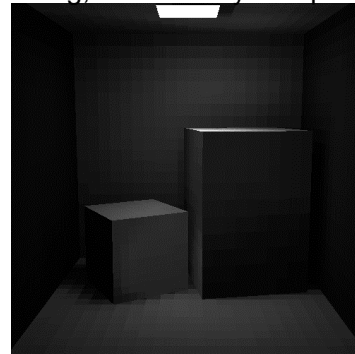


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## Antialiasing, No Intensity Interpolation

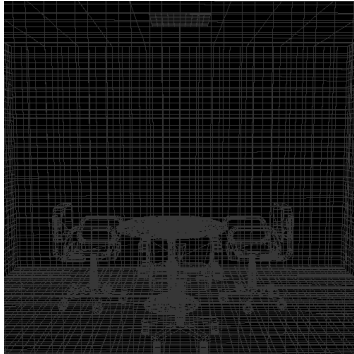


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Wireframe



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Classical, Resolution 300

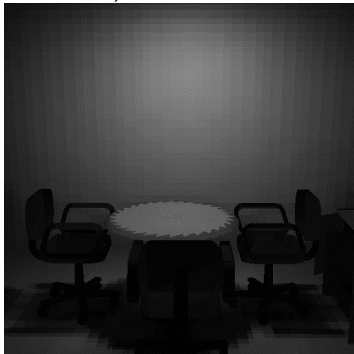


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Classical, Resolution 1200

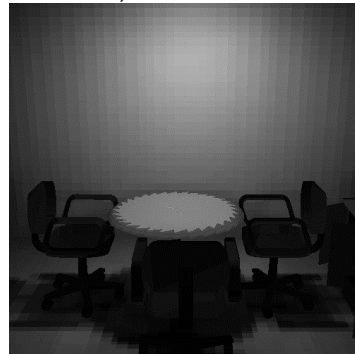


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Classical, Resolution 2500

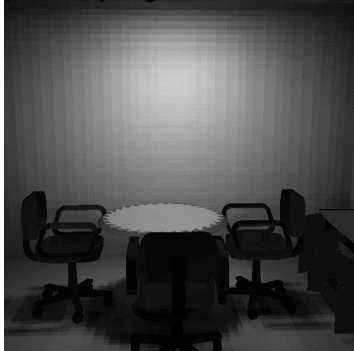


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Supersampling, Resolution 100

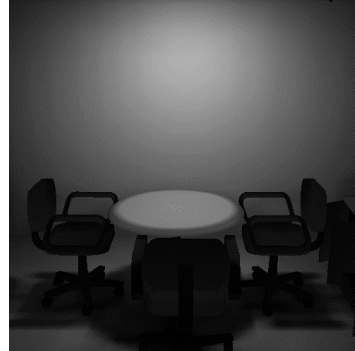


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Classical, Resolution 2500, Interpolated

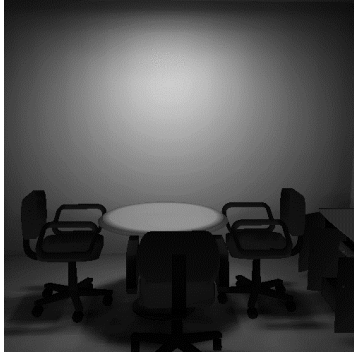


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## Supersampled, Res 100, Interpolated



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## Outline

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## Radiosity Equation Revisited

- Direct form

$$B_i = E_i + \rho_i \sum_j F_{ij} B_j$$

- As matrix equation

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & \rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & \rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

- Unknown: radiosity  $B_i$
- Known: emission  $E_i$ , form factor  $F_{ij}$ , reflect.  $\rho_i$

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## Classical Radiosity Algorithms

- Matrix Radiosity
  - Diagonally dominant matrix
  - Use Gauss-Seidel iterative solution
  - Time and space complexity is  $O(n^2)$  for  $n$  elements
  - Memory cost excessive
- Progressive Refinement Radiosity
  - Solve equations incrementally with form factors
  - Time complexity is  $O(n \cdot s)$  for  $s$  iterations
  - Used more commonly (space complexity  $O(n)$ )

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## Matrix Radiosity

- Compute all form factors  $F_{ij}$
- Make initial approximation to radiosity
  - Emitting elements  $B_i = E_i$
  - Other elements  $B_i = 0$
- Apply equation to get next approximation
 
$$B'_i = E_i + \rho_i \sum_j F_{ij} B_j$$
- Iterate with new approximation
- Intuitively
  - Gather incoming light for each element  $i$
  - Base new estimate on previous estimate

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## Radiosity Summary

- Assumptions
  - Opaque Lambertian surfaces (ideal diffuse)
  - Radiosity constant across each element
- Radiosity computation structure
  - Break scene into patches
  - Compute form factors between patches
    - Lighting independent
  - Solve linear radiosity equation
    - Viewer independent
  - Render using standard hardware

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## Lecture Summary

- Measures of Illumination
- The Radiosity Equation
- Form Factors
- Radiosity Algorithms

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## Preview

- Next Lecture
  - Radiosity refinements
  - Combining ray tracing and radiosity
- Assignment 7 (Ray Tracer) due April 23
- Different from OpenGL programming (150 pts)

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