

## 15-462 Computer Graphics I

### Lecture 21

# Visualization

Height Fields and Contours  
Scalar Fields  
Volume Rendering  
Vector Fields

[Angel Ch. 12]

April 23, 2002

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<http://www.cs.cmu.edu/~fp/courses/graphics/>

## Scientific Visualization

- Generally do not start with a 3D model
- Must deal with very large data sets
  - MRI, e.g.  $512 \times 512 \times 200 \approx 50\text{MB}$  points
  - Visible Human  $512 \times 512 \times 1734 \approx 433\text{ MB}$  points
- Visualize both real-world and simulation data
- User interaction
- Automatic search

## Types of Data

- Scalar fields (3D volume of scalars)
  - E.g., x-ray densities (MRI, CT scan)
- Vector fields (3D volume of vectors)
  - E.g., velocities in a wind tunnel
- Tensor fields (3D volume of tensors [matrices])
  - E.g., stresses in a mechanical part [Angel 12.7]
- Static or through time

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## Height Field

- Visualizing an explicit function

$$z = f(x,y)$$



- Adding contour curves

$$g(x,y) = c$$



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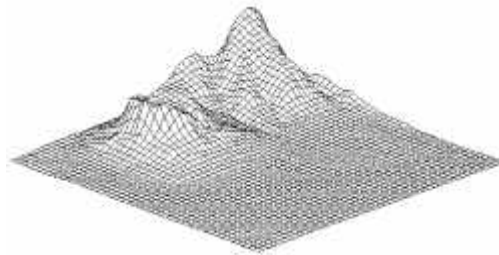
## Meshes

- Function is sampled (given) at  $x_i, y_i, 0 \leq i, j \leq n$
- Assume equally spaced

$$F(u, v) = c(u)c(v) \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} f(x, y) \cos \frac{(2x+1)u\pi}{2n} \cos \frac{(2y+1)v\pi}{2n}$$

where  $c(u) = 1/\sqrt{n}$  if  $u = 0$ ,  $c(u) = \sqrt{2/n}$  otherwise

- Generate quadrilateral or triangular mesh
- [Asst 1]



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## Contour Curves

- Recall: implicit curve  $f(x, y) = 0$
- $f(x, y) < 0$  inside,  $f(x, y) > 0$  outside
- Here: contour curve at  $f(x, y) = c$
- Sample at regular intervals for  $x, y$

$$x_i = x_0 + i\Delta x$$

$$y_j = y_0 + j\Delta y$$

- How can we draw the curve?

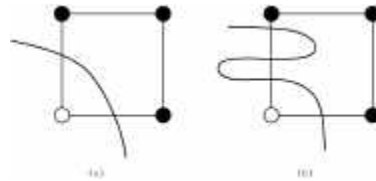
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## Marching Squares

- Sample function  $f$  at every grid point  $x_i, y_j$
- For every point  $f_{ij} = f(x_i, y_j)$  either  $f_{ij} \leq c$  or  $f_{ij} > c$
- Distinguish those cases for each corner  $x$ 
  - White:  $f_{ij} \leq c$
  - Black:  $f_{ij} > c$
- Now consider cases for curve
- Assume “smooth”
- Ignore  $f_{ij} = 0$



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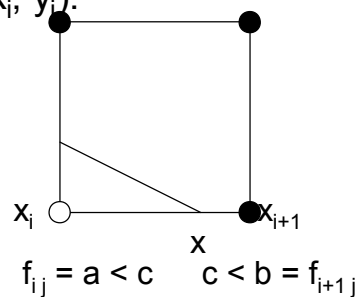
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## Interpolating Intersections

- Approximate intersection
  - Midpoint between  $x_i, x_{i+1}$  and  $y_j, y_{j+1}$
  - Better: interpolate
- If  $f_{ij} = a$  is closer to  $c$  than  $b = f_{i+1,j}$  then intersection is closer to  $(x_i, y_j)$ :

$$\frac{x - x_i}{x_{i+1} - x} = \frac{c - a}{b - c}$$

- Analogous calculation for  $y$  direction



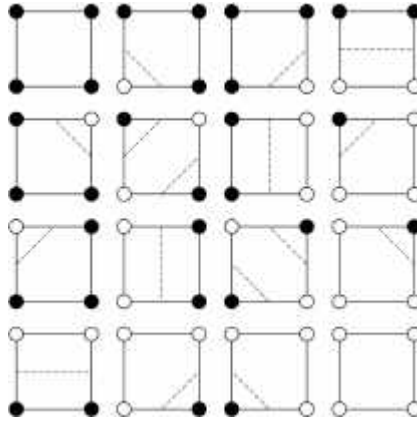
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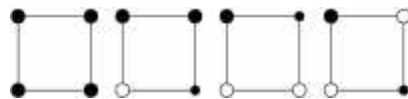
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## Cases for Vertex Labels

16 cases for vertex labels



4 unique mod. symmetries

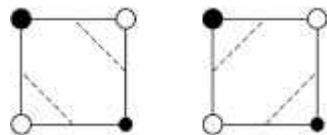


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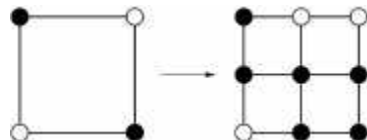
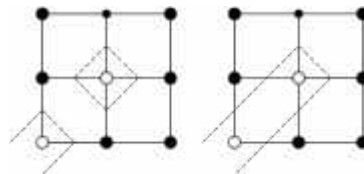
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## Ambiguities of Labelings



Ambiguous labels

Different resulting  
contours



Resolution by subdivision  
(where possible)

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## Marching Squares Examples

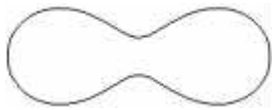
- Ovals of Cassini,  $50 \times 50$  grid

$$f(x, y) = (x^2 + y^2 + a^2)^2 - 4a^2x^2 - b^4$$

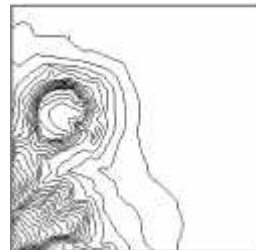
$$a = 0.49, b = 0.5$$



Midpoint



Interpolation



Contour plot of Honolulu data

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## Outline

- Height Fields and Contours
- Scalar Fields
- Volume Rendering
- Vector Fields

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## Scalar Fields

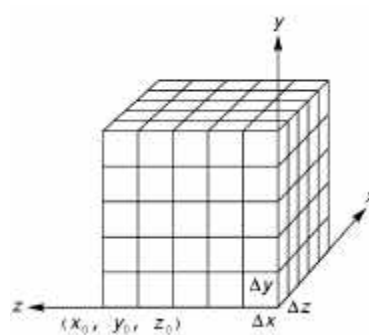
- Volumetric data sets
- Example: tissue density
- Assume again regularly sampled

$$x_i = x_0 + i\Delta x$$

$$y_j = y_0 + j\Delta y$$

$$z_k = z_0 + k\Delta z$$

- Represent as voxels



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## Isosurfaces

- $f(x,y,z)$  represents volumetric data set
- Two rendering methods
  - Isosurface rendering
  - Direct volume rendering (use all values [next])
- Isosurface given by  $f(x,y,z) = c$
- Recall implicit surface  $g(x, y, z)$ :
  - $g(x, y, z) < 0$  inside
  - $g(x, y, z) = 0$  surface
  - $g(x, y, z) > 0$  outside
- Generalize right-hand side from 0 to c

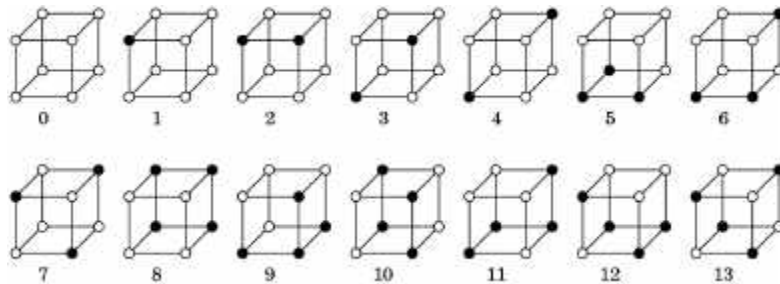
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## Marching Cubes

- Display technique for isosurfaces
- 3D version of marching squares
- 14 cube labelings (after elimination symmetries)



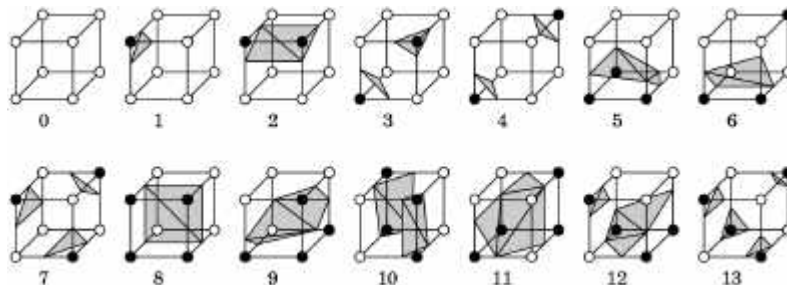
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## Marching Cube Tessellations

- Generalize marching squares, just more cases
- Interpolate as in 2D
- Ambiguities similar to 2D



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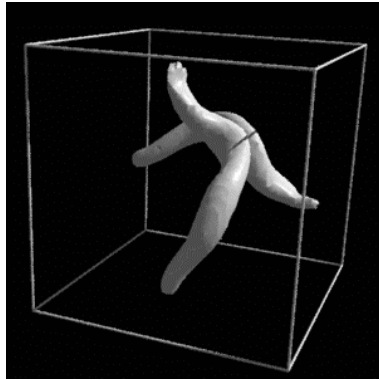
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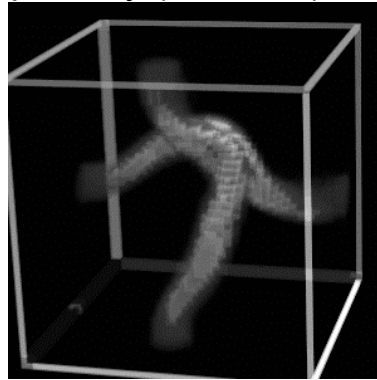


## Volume Rendering

- Sometimes isosurfaces are unnatural
- Use all voxels and transparency ( $\alpha$ -values)



Ray-traced isosurface



Volume rendering

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## Surface vs. Volume Rendering

- |                           |                            |
|---------------------------|----------------------------|
| • 3D model of surfaces    | • Scalar field in 3D       |
| • Convert to triangles    | • Convert to RGBA values   |
| • Draw primitives         | • Render volume “directly” |
| • Lose or disguise data   | • See data as given        |
| • Good for opaque objects | • Good for complex objects |

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## Sample Applications

- Medical
  - Computed Tomography (CT)
  - Magnetic Resonance Imaging (MRI)
  - Ultrasound
- Engineering and Science
  - Computational Fluid Dynamic (CFD)
  - Aerodynamic simulations
  - Meteorology
  - Astrophysics

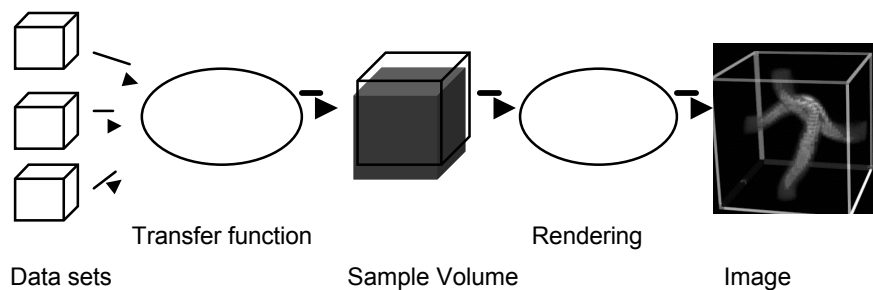
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## Volume Rendering Pipeline

- Transfer function: from data set to colors and opacities
  - Example:  $256 \times 256 \times 64 \times 2 = 4 \text{ MB}$
  - Example: use colormap (8 bit color, 8 bit opacity)



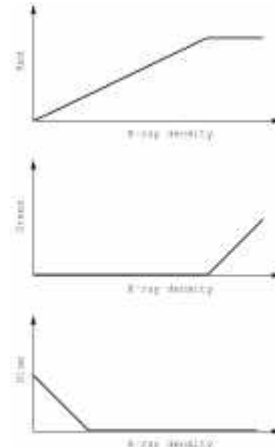
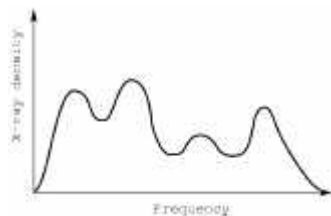
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## Transfer Functions

- Transform scalar data values to RGBA values
- Apply to every voxel in volume
- Highly application dependent
- Start from data histogram
- Opacity for emphasis



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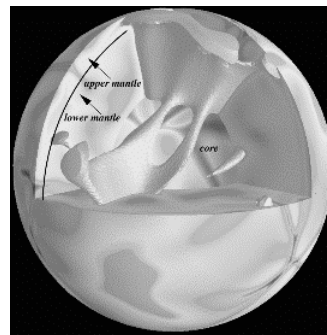
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## Transfer Function Example



Mantle Convection



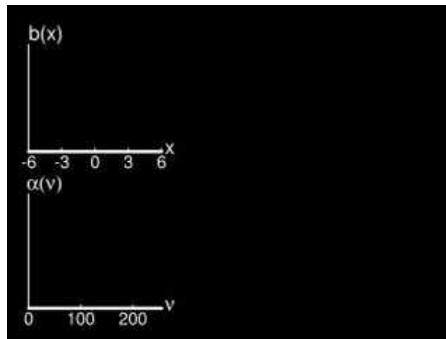
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## Transfer Function Example



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## Volume Ray Casting

- Three volume rendering techniques
  - Volume ray casting
  - Splatting
  - 3D texture mapping
- Ray Casting
  - Integrate color through volume
  - Consider lighting (surfaces?)
  - Use regular  $x,y,z$  data grid when possible
  - Finite elements when necessary (e.g., ultrasound)
  - 3D-rasterize geometrical primitives

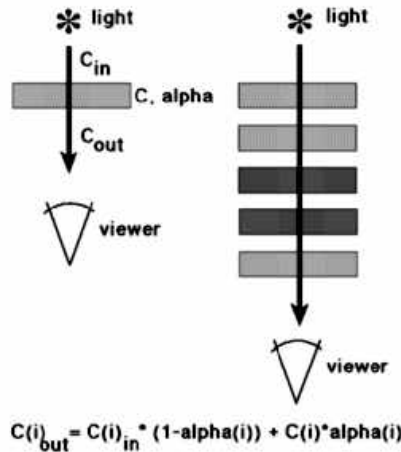
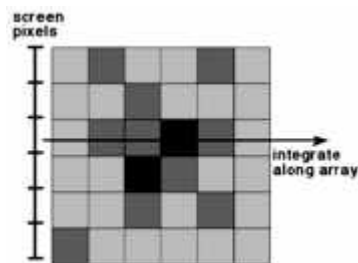
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## Accumulating Opacity

- $\alpha = 1.0$  is opaque
- Composit multiple layers according to opacity
- Use local gradient of opacity to detect surfaces for lighting



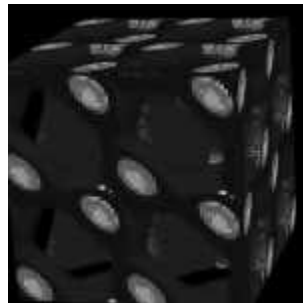
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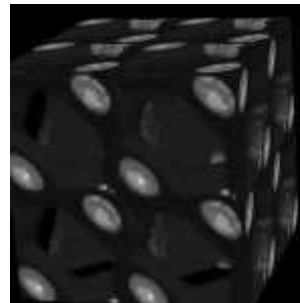
## Trilinear Interpolation

- Interpolate to compute RGBA away from grid
- Nearest neighbor yields blocky images
- Use trilinear interpolation
- 3D generalization of bilinear interpolation



Nearest neighbor

Trilinear interpolation



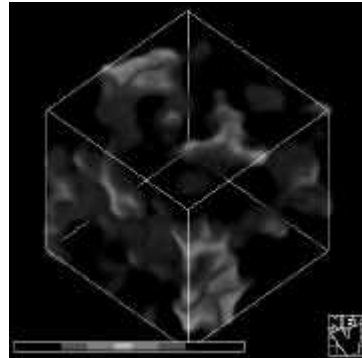
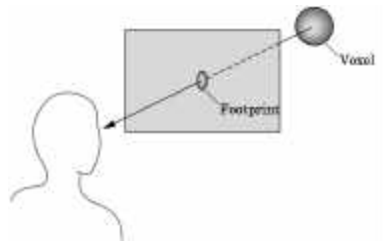
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## Splatting

- Alternative to ray tracing
- Assign shape to each voxel (e.g., Gaussian)
- Project onto image plane (splat)
- Draw voxels back-to-front
- Composite ( $\alpha$ -blend)



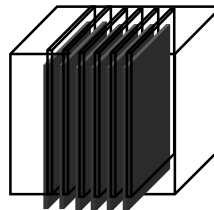
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## 3D Textures

- Alternative to ray tracing, splatting
- Build a 3D texture (including opacity)
- Draw a stack of polygons, back-to-front
- Efficient if supported in graphics hardware
- Few polygons, much texture memory



3D RGBA texture

Draw back to front

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## Example: 3D Textures



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## Example: 3D Textures

Emil Praun'01



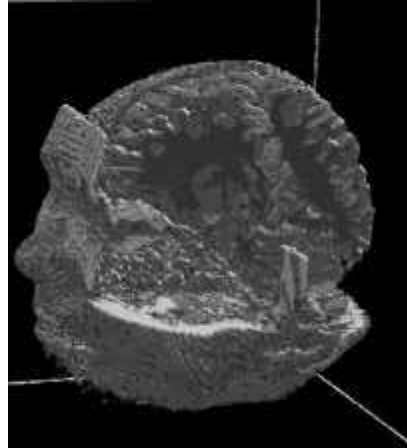
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## Other Techniques

- Use CSG for cut-away



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## Acceleration of Volume Rendering

- Basic problem: Huge data sets
- Program for locality (cache)
- Divide into multiple blocks if necessary
  - Example: marching cubes
- Use error measures to stop iteration
- Exploit parallelism

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## Outline

- Height Fields and Contours
- Scalar Fields
- Volume Rendering
- Vector Fields

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## Vector Fields

- Visualize vector at each  $(x,y,z)$  point
  - Example: velocity field
  - Example: hair
- Hedgehogs
  - Use 3D directed line segments (sample field)
  - Orientation and magnitude determined by vector
- Animation
  - Use for still image
  - Particle systems



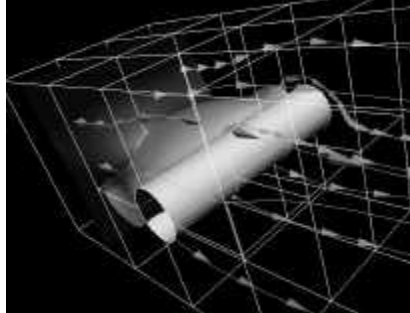
Blood flow in  
human carotid artery

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## Using Glyphs and Streaks



Glyphs for air flow



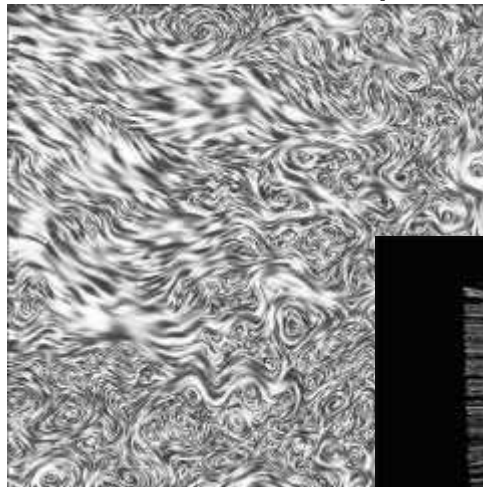
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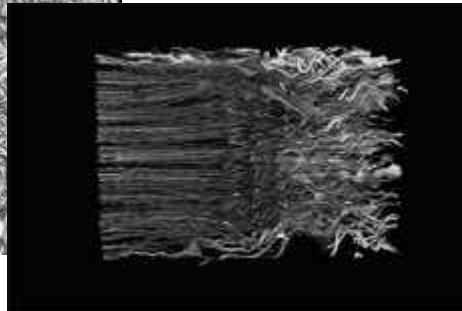
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## More Flow Examples



Banks and Interrante

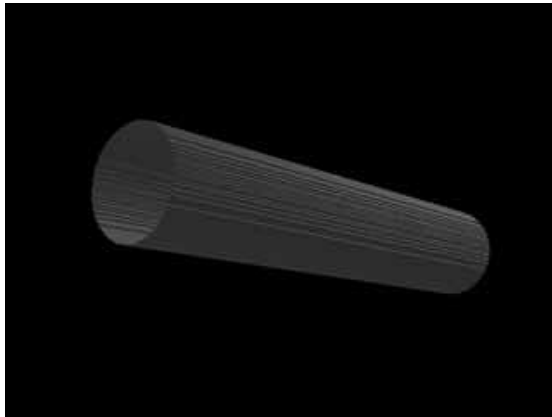


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## Example: Jet Shockwave



P. Sutton  
University of Utah

<http://www.sci.utah.edu/>

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## Summary

- Height Fields and Contours
- Scalar Fields
  - Isosurfaces
  - Marching cubes
- Volume Rendering
  - Volume ray tracing
  - Splatting
  - 3D Textures
- Vector Fields
  - Hedgehogs
  - Animated and interactive visualization

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## Preview

- Pick up Assignment 6
- Thursday
  - Non-photo-realistic rendering (NPR)
  - Assignment 7 (Ray Tracing) due!
  - Assignment 8 (written) out
- Next Tuesday
  - Guest lecture: Wayne Wooten, Pixar
- Next Thursday
  - Review for final
  - Assignment 8 due