# Substructural Typestates 

Filipe Militão (CMU \& UNL)
Jonathan Aldrich (CMU)
Luís Caires (UNL)

FACULDADE DE CIÊNCIAS E TECNOLOGIA UNIVERSIDADE NOVA DE LISBOA

## Motivation

File file = new File( "out.txt" ); file.write( "stuff" ); file.close(); file.write( "more stuff" );

## Motivation

File file = new File( "out.txt" ); file.write( "stuff" ); file.close();

## file.write( "more stuff");

FAILS with runtime exception ("invalid file descriptor")

## Motivation

```
class File {
    FileDescriptor fd;
    File( string filename ){
        fd = OS.createFile( filename );
    }
    void write( string s ){
    if( fd == null )
        throw Exception("invalid file descriptor");
    fd.write( s );
    }
    void close(){
    fd = null;
    }
}
```


## Motivation



## Motivation



## Motivation



## Contributions

I. Reconstruct typestate features from standard type-theoretic programming language primitives.
We focus on the following set of typestate features:
a) state abstraction, hiding an object representation while expressing the type of the state;
b) state "dimensions", enabling multiple orthogonal typestates over the same object;
c) "dynamic state tests", allowing a case analysis over the abstract state.
2. We show how to idiomatically support both state-based (typestate) and transition-based (behavioral types) specifications of abstract state evolution.

## Language

- Polymorphic $\boldsymbol{\lambda}$-calculus with mutable references (and immutable records, tagged sums, ...).
- Technically, we use a variant of $\mathbf{L}^{\mathbf{3}}$ adapted for usability (by simplifying the handling of capabilities, adding support for sum types, universal/existential type quantification, alternatives, labeled records, ...).

Ahmed, Fluet, and Morrisett. L³: A linear language with Iocations. Fundam. Inform. 2007.

## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.


## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.
ref $A$


## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities. - use location-dependent types to link both.



## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.



## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.



## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.



## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.



## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.


## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.

$$
\begin{gathered}
y: \operatorname{ref} p \\
z: \operatorname{ref} q \quad x: \operatorname{ref} p
\end{gathered}
$$

## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities. - use location-dependent types to link both.

$$
\begin{array}{ccc}
y: \text { ref } p & \text { rw } q B \\
z: \text { ref } q & x: \text { ref } p & \text { rw } p A
\end{array}
$$

## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.

$$
\begin{array}{ccc}
y: \text { ref } p & \text { ru } q B \\
z: \text { ref } q & x: \text { ref } p & \text { rw } p A
\end{array}
$$



## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.



## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.



## Language

- Mutable state handled as a linear resource: - split in pure references and linear capabilities.
- use location-dependent types to link both.



## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:

$$
\Gamma ; \Delta_{0} \vdash e: A \dashv \Delta_{1}
$$

## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:

Lexical Typing Environment

$$
\Gamma ; \Delta_{0} \vdash e: A \dashv \Delta_{1}
$$

## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:

Lexical Typing Environment

$$
\Gamma ; \Delta_{0}+e: A \nrightarrow \Delta_{1}
$$

Initial Linear Typing Environment

## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:


## Lexical Typing Environment

$$
\underbrace{\Gamma ;}_{\text {rTyping Environment }} \underbrace{A \dashv \Delta_{1}}_{\underbrace{}_{\text {Type of Expression }} \vdash e}
$$

## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:

Lexical Typing Environment
$\Gamma ; \Delta_{0}+e:$
Initial Linear Typing Environment

Resulting Effects


Type of Expression

## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:

$$
\underbrace{\Gamma: \Delta_{0} \vdash e: A+\Delta_{1}}_{\begin{array}{c}
\text { resources are } \\
\text { either consumed }
\end{array}}
$$

## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:

$$
\Gamma ; \Delta_{0} \vdash e: A \dashv \Delta_{1}
$$

## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:

$$
\underset{\substack{\text { r,threaded } \\ \text { through }}}{\Gamma ; \Delta_{0} \vdash e: A \dashv \Delta_{1}+e .}
$$

## Language

- Capabilities are (linear) typing artifacts (not values) that are threaded and stacked implicitly.
- For that, we use a Type-and-Effect system.
- Typing judgement format:

$$
\Gamma ; \Delta_{0} \vdash e: A+\Delta_{1}
$$

## Language

- Capabilities can be stacked and unstacked on top of some type, allowing them to accompany that type.


## Language

- Capabilities can be stacked and unstacked on top of some type, allowing them to accompany that type.
(т:Сар-Stack)
$\Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1}$
$\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}$


## Language

- Capabilities can be stacked and unstacked on top of some type, allowing them to accompany that type.
(т:Cap-Stack)
$\Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1}$
$\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}$


## Language

- Capabilities can be stacked and unstacked on top of some type, allowing them to accompany that type.
(т:Сар-Stack)
$\Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1}$
$\Gamma ; \Delta_{0}+e: A_{0}:: A_{1}+\Delta_{1}$


## Language

- Capabilities can be stacked and unstacked on top of some type, allowing them to accompany that type.
(т:CAP-Stack) (т:Cap-Unstack)
$\Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1} \quad \Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}$
$\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1} \quad \Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1}$


## Language

- Capabilities can be stacked and unstacked on top of some type, allowing them to accompany that type.
(т:Сар-Stack)
$\Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1}$
$\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}$


## Language

- Capabilities can be stacked and unstacked on top of some type, allowing them to accompany that type.



## Language

- Capabilities can be stacked and unstacked on top of some type, allowing them to accompany that type.
(т:CAP-Stack) (т:Cap-Unstack)
$\frac{\Gamma ; \Delta_{0}+e: A_{0} \dashv \Delta_{1}, A_{1}}{\Gamma ; \Delta_{0}+e: A_{0}:: A_{1}+\Delta_{1}} \frac{\Gamma ; \Delta_{0}+e: A_{0}:: A_{1} \dashv \Delta_{1}}{\Gamma ; \Delta_{0}+e: A_{0} \dashv \Delta_{1}, A_{1}}$
(т:Cap-Elim)

$$
\frac{\Gamma ; \Delta_{0}, x: A_{0}, A_{1}+e: A_{2} \dashv \Delta_{1}}{\Gamma ; \Delta_{0}, x: A_{0}:: A_{1} \vdash e: A_{2} \dashv \Delta_{1}}
$$

## Types

$A \quad:=$|  | $!A$ | (pure/persistent) |
| :--- | :--- | :--- |
|  | $A-A A$ | (linear function) |
|  | $A:: A$ | (stacking) |
|  | $A * A$ | (separation) |
|  | $X$ | (type variable) |
|  | $\forall X . A$ | (universal type quantification) |
|  | $\exists X . A$ | (existential type quantification) |
|  | $[\overline{\mathrm{f}: A}]$ | (record) |
|  | $\forall t . A$ | (universal location quantification) |
|  | $\exists t . A$ | (existential location quantification) |
|  | $\operatorname{ref} p$ | (reference type) |
|  | $\operatorname{rec} X . A$ | (recursive type) |
|  | $\sum_{i} 1_{i} \# A_{i}$ | (tagged sum) |
|  | $A \oplus A$ | (alternative) |
|  | $\operatorname{rw} p A$ | (read-write capability to $p$ ) |
|  | none | (empty capability) |

## s)

| $v \in$ Values | ::= | $\rho$ | (address) |
| :---: | :---: | :---: | :---: |
|  | \| | $x$ | (variable) |
|  | \| | fun( $x$ : A).e | (function) |
|  | \| | $\langle t\rangle e$ | (universal location) |
|  | \| | $\langle X\rangle e$ | (universal type) |
|  | \| | $\langle p, v\rangle$ | (pack location) |
|  | \| | $\langle A, v\rangle$ | (pack type) |
|  | \| | $\{\overline{f=v}\}$ | (record) |
|  | \| | $1 \# v$ | (tagged value) |
| $e \in$ Exprs | ::= | $v$ | (value) |
|  | \| | $\nu[p]$ | (location application) |
|  | \| | $\nu[A]$ | (type application) |
|  | \| | $v . f$ | (field) |
|  | \| | $v \nu$ | (application) |
|  | \| | let $x=e$ in $e$ end | (let) |
|  | I | open $\langle t, x\rangle=v$ in $e$ end | (open location) |
|  | \| | open $\langle X, x\rangle=v$ in $e$ end | (open type) |
|  | \| | new $v$ | (cell creation) |
|  | \| | delete $v$ | (cell deletion) |
|  | \| | ! $v$ | (dereference) |
|  | \| | $v:=v$ | (assign) |
|  | 1 | case $v$ of $\overline{1 \# x \rightarrow e}$ end | (case) |

## SHAM2N

| $v \in$ Values $\quad::=$ | $\rho$ | (address) |
| :---: | :---: | :---: |
| \| | $x$ | (variable) |
| \| | fun( $x$ : A).e | (function) |
| \| | $\langle t\rangle e$ | (universal location) |
| \| | $\langle X\rangle e$ | (universal type) |
| \| | $\langle p, v\rangle$ | (pack location) |
| \| | $\langle A, v\rangle$ | (pack type) |
| \| | $\{\overline{f=v}\}$ | (record) |
| \| | $1 \# v$ | (tagged value) |
| $e \in$ Exprs. : $:=$ | $v$ | (value) |
| \| | $v[p]$ | (location application) |
| I | $v[A]$ | (type application) |
|  | $v . f$ | (field) |
| let-expanded | $v v$ | (application) |
| let-expanded | let $x=e$ in $e$ end | (let) |
|  | open $\langle t, x\rangle=v$ in $e$ end | (open location) |
|  | open $\langle X, x\rangle=v$ in $e$ end | (open type) |
| \| | new $v$ | (cell creation) |
| \| | delete $v$ | (cell deletion) |
| \| | !v | (dereference) |
| I | $v:=v$ | (assign) |
| \| | case $v$ of $\overline{1 \# x \rightarrow e}$ end | (case) |

## s)

| $v \in$ Values | $\rho$ | (address) |
| :---: | :---: | :---: |
|  | $x$ | (variable) |
|  | fun( $x$ : A).e | (function) |
|  | $\langle t\rangle e$ | (universal location) |
|  | $\langle X\rangle e$ | (universal type) |
|  | $\langle p, v\rangle$ | (pack location) |
|  | $\langle A, v\rangle$ | (pack type) |
|  | $\{\overline{f=v}\}$ | (record) |
|  | l\#v | (tagged value) |
| $e \in$ Exprs. | $v$ | (value) |
|  | $\nu[p]$ | (location application) |
|  | $\nu[A]$ | (type application) |
| let-expanded | $v . f$ | (field) |
|  | $v v$ | (application) |
|  | let $x=e$ in $e$ end | (let) |
|  | open $\langle t, x\rangle=v$ in $e$ end | (open location) |
|  | open $\langle X, x\rangle=v$ in $e$ end | (open type) |
|  | new $v$ | (cell creation) |
|  | delete $v$ | (cell deletion) |
|  | ! $v$ | (dereference) |
|  | $v:=v$ | (assign) |
|  | case $v$ of $\overline{1 \# x \rightarrow e}$ end | (case) |

## Pair Example

- Function that creates stateful Pair objects.
- The Pair's components (left and right) are private, not accessible to clients.
- The state of Pair is changed indirectly by calling functions contained in a labeled record (which are technically closures).

```
let newPair = fun( _ : [] ).
open \(\langle p l, l>=\) new \(\overline{\{ }\}\) in
open <pr,r> = new \{\} in
    \{
        initL = fun( i : int : : rw pl [] ). l := i,
        initR = fun( i : int : : rw pr [] ). r := i,
        sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
        destroy = fun( _ : [] :: rw pl int * rw pr int ).
                            delete \(l\); delete \(r\)
        \(\}\)
    end
    end
```

```
let newPair = fun( : [] ).
open <pl,l> = new {} in
open <pr,r> = new {} in
    {
        initL = fun( i : int :: rw pl [] ). l := i,
        initR = fun( i : int :: rw pr [] ). r := i,
        sum = fun( : [] :: rw pl int * rw pr int ). !l+!r,
        destroy = fun( : [] :: rw pl int * rw pr int ).
                        delete l; delete r
        }
end
end
```

```
let newPair = fun( : [] ).
open <pl,l> = new {} in
open <pr,r> = new {} in
    {
    initL = fun( i : int :: rw pl [] ). l := i,
    initR = fun( i : int :: rw pr [] ). r := i,
    sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
    destroy = fün( _ : [] :: rw pl int * rw pr int ).
                        delete l; delete r
    }
    end
    end
```

(т:NEw)

## (т:Loc-Open)

$$
\Gamma ; \Delta_{0} \vdash v: A+\Delta_{1} \quad \Gamma, t: \text { loc; } \Delta_{1}, x: A_{0}+e: A_{1}+\Delta_{2}
$$

$\overline{\left.\Gamma ; \Delta_{0} \vdash \text { new } v: \exists t \text {.(ref } t:: \text { rw } t A\right) \dashv \Delta_{1}} \overline{\Gamma ; \Delta_{0} \vdash \text { open }\langle t, x\rangle=v \text { in } e \text { end }: A_{1} \dashv \Delta_{2}}$

```
let newPair = fun( _ [] ). \Gamma = pl : loc, l : ref pl
open <pl,l> = new {} in \Delta = rw pl []
open <pr,r> = new {} in
    {
    initL = fun( i : int :: rw pl [] ). l := i,
    initR = fun( i : int :: rw pr [] ). r := i,
    sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
    destroy = fun( _ : [] :: rw pl int * rw pr int ).
                        delete l; delete r
}
end
end
```

(т:New)
(T:Loc-Open)

$$
\Gamma ; \Delta_{0} \vdash v: \exists t \cdot A_{0} \dashv \Delta_{1}
$$

$$
\Gamma ; \Delta_{0} \vdash v: A \nmid \Delta_{1} \quad \Gamma, t: \text { loc; } \Delta_{1}, x: A_{0}+e: A_{1}+\Delta_{2}
$$

$\overline{\left.\Gamma ; \Delta_{0} \vdash \text { new } v: \exists t \text {.(ref } t:: \text { rw } t A\right) \dashv \Delta_{1}} \overline{\Gamma ; \Delta_{0} \vdash \text { open }\langle t, x\rangle=v \text { in } e \text { end }: A_{1} \dashv \Delta_{2}}$

```
\Gamma = pl : loc, l : ref pl,
    pr : loc, r : ref pr
\Delta = rw pl [], rw pr []
open <pr,r> = new {} in
    initL = fun( i : int :: rw pl [] ). l := i,
    initR = fun( i : int :: rw pr [] ). r := i,
    sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
    destroy = fun( _ : [] :: rw pl int * rw pr int ).
                        delete l; delete r
        }
    end
    end
```

(T:NEW)
(T:Loc-Open)

$$
\Gamma ; \Delta_{0} \vdash v: \exists t \cdot A_{0} \dashv \Delta_{1}
$$

$$
\Gamma ; \Delta_{0} \vdash v: A \dashv \Delta_{1} \quad \Gamma, t: \text { loc; } \Delta_{1}, x: A_{0} \vdash e: A_{1}+\Delta_{2}
$$

$\overline{\left.\Gamma ; \Delta_{0} \vdash \text { new } v: \exists t \text {.(ref } t:: \text { rw } t A\right) \dashv \Delta_{1}} \overline{\Gamma ; \Delta_{0} \vdash \text { open }\langle t, x\rangle=v \text { in } e \text { end }: A_{1} \dashv \Delta_{2}}$

$$
\begin{aligned}
& \Gamma=p l: l \text { lock, l : ref pl, } \\
& p r: \text { lock, r : ref pr } \\
& \Delta= \text { rb pl [], rw pr [] }
\end{aligned}
$$

open <pr,r> = new \{\} in
\{
init L = fun( i : int : : pw pl [] ). l := i, init $=$ fun( $i \quad$ : int : : pw pr [] ). r := i, sum $=$ fun( _ : [] :: rx pl int * pw pr int ). !l+!r, destroy $=$ fun( _ : [] :: pw pl int * pw pr int ).
delete $l$; delete $r$
\}
end
end
(т:Function)

$$
\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv
$$

$$
\overline{\Gamma ; \Delta \vdash v: A \dashv}
$$

$$
\overline{\Gamma ; \Delta \vdash\{\overline{f=v}\}:[\overline{f: A}] \dashv .}
$$

$\overline{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv} \overline{\Gamma ; \Delta \vdash\{\overline{\mathrm{f}=v}\}:[\overline{\mathrm{f}: A}] \dashv .}$

```
let newPair = fun( _ : [] ).
    open <pl,l> = new {}{}\mathrm{ in
    open <pr,r> = new {} in
        {
        initL = fun( i : int :: rw pl [] ). l := i,
        initR = fun( i : int :: rw pr [] ). r := i,
        sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
        destroy = fun( _ : [] :: rw pl int * rw pr int ).
                            delete l; delete r
        }
    end
    end
```

```
let newPair = fun( _ : [] ).
open <pl,l> = new {}{}\mathrm{ in
open <pr,r> = new {} in
{
    InitL = fun( i : int :: rw pl [] ). l := i,
    initR = fun( i : int :: rw pr [] ). r := i,
    sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
    destroy = fun( _ : [] :: rw pl int * rw pr int ).
                        delete l; delete r
        }
    end
    end
```

(т:Function)
$\frac{\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv \cdot}{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv}$
(т:Сap-Stack)
$\frac{\Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1}}{\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}}$

```
let newPair = fun(
open <pl,l> = new {}{
open <pr,r> = new {}:\Delta = rw pl []
```

\{
initL $=$ fun( i : int : : rw pl [] ). l := i,
initR = fun( i : int :: rw pr [] ). r : $=\mathrm{i}$,
sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
destroy $=$ fun( $\quad$ : [] :: rw pl int * rw pr int ).
delete l; delete r
\}
end
end
(т:Function)
$\frac{\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv \cdot}{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv \cdot}$
(т:Сap-Stack)
$\frac{\Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1}}{\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}}$

```
let newPair = fun( _ : [] ).
```

```
open <pl,l> = new {}{}\mathrm{ in
open <pr,r> = new {} in
\Delta = rw pl int
```

    initL \(=\) fun( i : int : : rw pl [] ). l := i,
    initR = fun( i : int :: rw pr [] ). r := i,
    sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
    destroy \(=\) fün( _ : [] :: rw pl int * rw pr int ).
                                    delete l; delete r
    \}
end
end
(т:Function)
$\frac{\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv \cdot}{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv \cdot}$
(т:Сар-Stack)
$\frac{\Gamma ; \Delta_{0}+e: A_{0} \dashv \Delta_{1}, A_{1}}{\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}}$

```
let newPair = fun( _ : [] ).
open <pl,l> = new {}{}\mathrm{ in
open <pr,r> = new {} in
```

\{
initL $=$ fun( i : int : : rw pl [] ). l := i,

delete l; delete r
\}
end
end
(T:FUNCTION)
$\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv$.
$\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv$.
(т:Сар-Sтаск)
$\frac{\Gamma ; \Delta_{0}+e: A_{0} \dashv \Delta_{1}, A_{1}}{\Gamma ; \Delta_{0}+e: A_{0}:: A_{1} \dashv \Delta_{1}}$

## (т:Pure)

```
```

let newPair = fun( _ : [] ).

```
```

let newPair = fun( _ : [] ).
open <pl,l> = new {} in
open <pl,l> = new {} in
open <pr,r> = new {} in

```
```

open <pr,r> = new {} in

```
```

$\Gamma ; \cdot \vdash v: A \dashv \cdot$
$\overline{\Gamma ; \vdash v:!A \dashv}$
initL $=$ fun( i : int :: rw pl [] ). l := i,

delete l; delete r
\{
\}
end
end
(т:Function)
$\frac{\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv \cdot}{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv \cdot}$
(т:Сар-Stack)
$\frac{\Gamma ; \Delta_{0} \vdash e: A_{0} \dashv \Delta_{1}, A_{1}}{\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}}$

```
let newPair = fun(
    open <pl,l> = new {} in
    open <pr,r> = new {} in
    {
    initL = fun( i : int :: rw pl [] ). l := i,
    initR = fun( i : int :: rw pr [] ). r := i,
    sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
    destroy = fün( _ : [] :: rw pl int * rw pr int ).
                                    delete l; delete r
}
end
[
    initL : !( ( int :: rw pl [] ) \multimap ( [] :: rw pl int ) ),
    initR : !( ( int :: rw pr [] ) \multimap- ( [] :: rw pr int ) ),
        sum : !( ( [] :: rw pl int * rw pr int ) }
                                (int :: rw pl int * rw pr int ) ),
    destroy : !( ( [] :: rw pl int * rw pr int ) \multimap [] )
    ]
```

let newPair $=$ fun( $:[]$ ).
open <pl,l> = new \{\} in
open <pro> = new \{\} ~ i n ~ $\Delta=\operatorname{rW}$ pl [], rh pr []
\{

```
initL = fun( i : int :: rw pl [] ). l := i,
```

init = fun( i : int :: pw pr [] ). r := i,
sum = fun( _ : [] :: rb pl int * pw pr int ). !l+!r, destroy $=$ fun( _ : [] : : rw pl int $*$ pw pr int ).
delete $l$; delete $r$
\}
end
end

```
[
    initL : !( ( int :: rw pl [] ) \multimap ( [] :: rw pl int ) ),
    initR : !( ( int :: rw pr [] ) \multimap- ( [] :: rw pr int ) ),
        sum : !( ( [] :: rw pl int * rw pr int ) -o 
                                (int :: rw pl int * rw pr int ) ),
    destroy : !( ( [] :: rw pl int * rw pr int ) \multimap [] )
    ]
```

```
let newPair = fun(
    open <pl,l> = new {} in
    open <pr,r> = new {} in
    {
    initL = fun( i : int :: rw pl [] ). l := i,
    initR = fun( i : int :: rw pr [] ). r := i,
    sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
    destroy = fün( _ : [] :: rw pl int * rw pr int ).
    delete l; delete r
    }
    end
        [
            initL : !( ( int :: rw pl [] ) \multimap ( [] :: rw pl int ) ),
            initR : !( ( int :: rw pr [] ) \multimap- ( [] :: rw pr int ) ),
            sum : !( ( [] :: rw pl int * rw pr int ) -
                                (int :: rw pl int * rw pr int ) ),
    destroy : !( ( [] :: rw pl int * rw pr int ) \multimap [] )
    ] :: ( rw pl [] * rw pr [] )
```


## let newPair = fun( <br> open <pl,l> $=$ new $\}$ in

open <pr,r> = new \{\} in
<pl, <pr, \{
init = fun( i : int :: pw pl [] ). l := i, init $=$ fun( $i$ : int :: pw pr [] ). r := i, sum = fun( $: ~[] ~:: ~ r w ~ p l ~ i n t ~ * ~ r w ~ p r ~ i n t ~) . ~!l+!r, ~$ destroy $=$ fun( _ : [] : : rw pl int $*$ pw pr int ).
delete $l$; delete $r$


## let newPair = fun(

open $<p l$, l> $=$ new $\}$ in
open <pro> = new \{\} ~ i n ~
<pl, <pr, \{
init $=$ fun( i : int :: pw pl [] ). l := i, init = fun( i : int :: pw pr [] ). r := i, sum = fun( _ : [] :: rv pl int * pw pr int ). !l+!r, destroy $=$ fun( _ : [] : : rw pl int $*$ pw pr int ).
delete $l$; delete $r$


```
let newPair = fun(
    open <pl,l> = new {} in
    open <pr,r> = new {} in
    < rw pl [], < rw pl int, < rw pr [], < rw pr int {
        initL = fun( i : int :: rw pl [] ). l := i,
        initR = fun( i : int :: rw pr [] ). r := i,
        sum = fun( _ : [] :: rw pl int * rw pr int ). !l+!r,
        destroy = fun( _ : [] :: rw pl int * rw pr int ).
                        delete l; delete r
}> > > >
    end
    end
```

```
#EL.GL.GER.GR.( [
```

\#EL.GL.GER.GR.( [
initL : !( int :: EL — [] :: L ),
initL : !( int :: EL — [] :: L ),
initR : !( int :: ER — [] :: R ),
initR : !( int :: ER — [] :: R ),
sum : !( [] :: L * R \multimap int :: L * R ),
sum : !( [] :: L * R \multimap int :: L * R ),
destroy : !( [] :: L * R \multimap [] )
destroy : !( [] :: L * R \multimap [] )
] :: EL * ER )

```
    ] :: EL * ER )
```


## Pair Typestate

newPair ：

$$
\begin{aligned}
& \text { !( [] ○ ヨEL.ヨL.ヨER.ヨR.([ } \\
& \text { initL : ! ( int :: EL } \quad \text { [] :: L ), } \\
& \text { initR : !( int :: ER } \quad \text { [] :: R ), } \\
& \text { sum : ! ( [] :: L * } \quad \rightarrow \text { int :: } L \text { * } R \text { ), } \\
& \text { destroy : ! ( [] :: L * R } \rightarrow \text { [] ) } \\
& \text { ] :: EL * ER ) ) }
\end{aligned}
$$

－Type expresses the changing properties of the object＇s state， typestate（EmptyLeft，Left，EmptyRight and Right）．
－Orthogonal typestates，＂state dimensions＂（EL／L and $E R / R$ ）， correlate to separate internal state that operates independently．

## Stack Typestate

- Type of a function (polymorphic in the contents to be stored in the stack) that creates stack objects.
- Each stack has two states: Empty and NonEmpty.
- Imprecision in the exact state of the stack is typed with $\mathrm{E} \oplus \mathrm{NE}$ (alternative): we either have the E typestate or NE the typestate.


## Stack Typestate

- Type of a function (polymorphic in the contents to be stored in the stack) that creates stack objects.
- Each stack has two states: Empty and NonEmpty.
- Imprecision in the exact state of the stack is typed with $\mathrm{E} \oplus \mathrm{NE}$ (alternative): we either have the E typestate or NE the typestate.

Typestates do not exist at runtime. How can client code distinguish between different states without breaking the abstraction?

```
newStack :
    \forallT.( [] -
\existsE.\existsNE.[
    push : T :: E\oplusNE \multimap [] :: NE,
    pop : [] :: NE \multimap T :: E\oplusNE,
isEmpty : [] :: E\oplusNE \multimap Empty#([]::E) + NonEmpty#([]::NE),
    del : [] :: E -o []
] :: E )
```

Note: !'s omitted from the type for brevity.

## newStack :

```
\forallT.( [] —
    \existsE.\existsNE.[
    push : T :: E\oplusNE \multimap [] :: NE,
    pop : [] :: NE }-\textrm{T}:: E\oplusN\mathbb{NE,
    isEmpty : [] :: E\oplusNE \multimap Empty#([]::E) + NonEmpty#([]::NE),
    del : [] :: E \multimap []
    ] :: E )
```

Note: !'s omitted from the type for brevity.

## newStack :



Note: !'s omitted from the type for brevity.

## Contributions

I. Reconstruct typestate features from standard type-theoretic programming language primitives. We focus on the following set of typestate features:
a) state abstraction, hiding an object representation while expressing the type of the state;
b) state "dimensions", enabling multiple orthogonal typestates over the same object;
c) "dynamic state tests", allowing a case analysis over the abstract state.
2. We show how to idiomatically support both state-based (typestate) and transition-based (behavioral types) specifications of abstract state evolution.

## Back to Pair...

The evolution of the abstract state can be specified using a state-machine/automaton/protocol.


## Back to Pair...

The evolution of the abstract state can be specified using a state-machine/automaton/protocol.


Typestates focus on the states that model the abstracted changes of the mutable state.

## Back to Pair...

The evolution of the abstract state can be specified using a state-machine/automaton/protocol.


Behavioral Types focus on the transitions ("behavior") keeping the states anonymous.

# Abstracting and Hiding State 

- In our system, the notion of typestates is related to state abstraction, while the notion of behavior is related to hiding state.
- With typestates, states are named which can be convenient when there are multiple paths through the protocol.
- With behavioral types, states are implicit which simplifies descriptions of linear usages and makes it easier to provide structural equivalences.

Caires and Seco. The type discipline of behavioral separation. POPL 2013.

## Abstracting and Hiding State

- We have already seen how to model typestates through standard existential abstraction.
- Interestingly, the notion of "behavior" can be modeled with what was already shown!
- However, it requires using an idiom to capture the typestate inside a function effectively hiding it.


## Borrowing and Capturing

- A typestate can be borrowed by a function if that function requires the typestate as an argument but the function returns the typestate as a result.
initL : !( int :: EL ↔ [] :: L )


# Borrowing and Capturing 

- Alternatively, a function may depend on state that was captured from the enclosing linear environment (similar to a closure, but with state).
(т:Function)

$$
\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv
$$

$\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) . e: A_{0} \multimap A_{1} \dashv$.

# Borrowing and Capturing 

Alternatively, a function may depend on state that was captured from the enclosing linear environment (similar to a closure, but with state).

```
fun( x : int ).(initL x)
```

(т:Function)

$$
\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv
$$

$\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) . e: A_{0} \multimap A_{1} \dashv \cdot$

# Borrowing and Capturing 

- Alternatively, a function may depend on state that was captured from the enclosing linear environment (similar to a closure, but with state).

$$
\text { 「 = initL : ! ( int :: EL } \multimap[] \text { :: L ) }
$$

fun( x : int ).(initL x$)$
(т:Function)

$$
\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv
$$

$\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv$.

# Borrowing and Capturing 

- Alternatively, a function may depend on state that was captured from the enclosing linear environment (similar to a closure, but with state).

$$
\text { 「 = initL : !( int :: EL } \multimap[] \text { :: L ) }
$$

$\Delta=E L$ fun( x : int ).(initL x$)$
(т:Function)

$$
\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv
$$

$\overline{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv .}$

# Borrowing and Capturing 

- Alternatively, a function may depend on state that was captured from the enclosing linear environment (similar to a closure, but with state).
「 = initL : ! ( int : : EL $\multimap$ [] :: L )
$\Delta=E L$ fun( $x$ : int ).(initL $x)$
(т:Function)
$\frac{\Gamma ; \Delta, x: A_{0}+e: A_{1} \dashv \cdot}{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv .}$
(т:Cap-Stack)
$\frac{\Gamma ; \Delta_{0}+e: A_{0}+\Delta_{1}, A_{1}}{\Gamma ; \Delta_{0}+e: A_{0}:: A_{1} \dashv \Delta_{1}}$


# Borrowing and Capturing 

- Alternatively, a function may depend on state that was captured from the enclosing linear environment (similar to a closure, but with state).

$$
\text { 「 = initL : !( int :: EL } \multimap[] \text { :: L ) }
$$

$\Delta=E L$ fun( $x$ : int ).(initL $x) \Delta=$.
(т:Function)
$\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv$.
$\overline{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) . e: A_{0} \multimap A_{1} \dashv .}$
(т:САР-Stack)

$$
\frac{\Gamma ; \Delta_{0}+e: A_{0}+\Delta_{1}, A_{1}}{\Gamma ; \Delta_{0}+e: A_{0}:: A_{1}+\Delta_{1}}
$$

$\frac{\Gamma ; \Delta, x: A_{0}+e: A_{1} \dashv \cdot}{\Gamma ; \Delta \vdash \operatorname{fun}\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv \cdot} \quad \frac{\Gamma ; \Delta_{0}+e: A_{0} \dashv \Delta_{1}, A_{1}}{\Gamma ; \Delta_{0}+e: A_{0}:: A_{1} \dashv \Delta_{1}}$

# Borrowing and Capturing 

- Alternatively, a function may depend on state that was captured from the enclosing linear environment (similar to a closure, but with state).

$$
\text { 「 = initL : !( int :: EL } \multimap[] \text { :: L ) }
$$

$\Delta=E L$ fun( x : int ).(initL x$) \quad \Delta=$.

$$
\text { int } \rightarrow[]:: L
$$

| (т:Function) <br> $\Gamma ; \Delta, x: A_{0} \vdash e: A_{1} \dashv$ | (т:CAP-StaCK) <br> $\Gamma ; \Delta \vdash$ fun $\left(x: A_{0}\right) \cdot e: A_{0} \multimap A_{1} \dashv \cdot$ |
| :--- | :--- |
| $\frac{\Gamma ; \Delta_{0}+e: A_{0} \dashv \Delta_{1}, A_{1}}{\Gamma ; \Delta_{0} \vdash e: A_{0}:: A_{1} \dashv \Delta_{1}}$ |  |

## Hiding (Type)state

- Capturing the typestate enables us to hide the typestate needed by the function's argument.
- Hiding the typestate from the result is not immediately possible. However, we can define a complete sequence of uses ("behavior") that ends in a function that destroys the (type)state.
- One possible linear "behavior" for the Pair is:



## Hiding (Type)state

- Capturing the typestate enables us to hide the typestate needed by the function's argument.
- Hiding the typestate from the result is not immediately possible. However, we can define a complete sequence of uses ("behavior") that ends in a function that destroys the (type)state.
- One possible linear "behavior" for the Pair is:


```
fun( a : int ).
    {
    initR(a)
    fun( b : int ).
        {
        initL(b)
        fun( _ : [] ).
            {
                sum(_)
                fun( _ : [] ).destroy(_)
            }
    }
}
```

```
fun( a : int ).
    {
    initR(a)
```

    fun( b : int ).
        \{
    initL(b)
    fun( _ : [] ).
sum(_)

fun( a : int ).

## initR(a)

```
fun( b : int ).
    {
        initL(b)
```

        fun( _ : [] ).
        \{um (_) [] ::L*R int \(:: L * R\)
    
fun( a : int ).
\{

,

$$
\text { fun( } b \text { : int ). }
$$

\{

$$
\operatorname{initL}(\mathrm{b})<{ }^{\text {int }:: E L \multimap[]:: L}
$$

,
fun( _ : [] ).


## $\Delta=E L, E R$

fun( a : int ).
\{

,

```
fun( b : int ).
```

    \{
        initL(b) int : : EL \(\multimap[]:: L\)
    ,
            fun( _ : [] ).
    



Clients never see the underlying typestates. They only see the usage requirement ("behavior").

## Technical Results

Theorem 1 (Progress). If $e_{0}$ is a closed expression (and where $\Gamma$ and $\Delta_{0}$ are also closed) such that:

$$
\Gamma ; \Delta_{0} \vdash e_{0}: A \dashv \Delta_{1}
$$

then either:

- $e_{0}$ is a value, or;
- if exists $H_{0}$ such that $\Gamma ; \Delta_{0} \vdash H_{0}$ then $\left\langle H_{0} \| e_{0}\right\rangle \mapsto\left\langle H_{1} \| e_{1}\right\rangle$.

Theorem 2 (Preservation). If $e_{0}$ is a closed expression such that:

$$
\Gamma_{0} ; \Delta_{0} \vdash e_{0}: A \dashv \Delta \quad \Gamma_{0} ; \Delta_{0} \vdash H_{0} \quad\left\langle H_{0} \| e_{0}\right\rangle \mapsto\left\langle H_{1} \| e_{1}\right\rangle
$$

then, for some $\Delta_{1}, \Gamma_{1}$ :

$$
\Gamma_{0}, \Gamma_{1} ; \Delta_{1} \vdash H_{1} \quad \Gamma_{0}, \Gamma_{1} ; \Delta_{1} \vdash e_{1}: A \dashv \Delta
$$

## Related Work

DeLine and Fähndrich. Typestates for objects. ECOOP 2004.
DeLine and Fähndrich. Enforcing high-level protocols in low-level software. PLDI 2001.

Bierhoff and Aldrich. Modular typestate checking of aliased objects. OOPSLA 2007.

Beckman, Bierhoff, and Aldrich. Verifying correct usage of atomic blocks and typestate. OOPSLA 2008.

Sunshine, Naden, Stork, Aldrich, and Tanter. First-class state change in Plaid. OOPSLA 201I.

- They support many advanced uses (method dispatch, inheritance, sharing mechanisms, concurrency, etc).
- We focus on reconstructing a smaller set of typestate features from type-theoretic primitives (separation and linear logic). Which enables combining abstracting and hiding state.


## Related Work

Ahmed, Fluet, and Morrisett. L3: A linear language with locations. Fundam. Inform. 2007.

Walker and Morrisett. Alias types for recursive data structures.TIC 200I.
Smith,Walker, and Morrisett. Alias types. ESOP 2000.

- We extend their work with usability related changes (implicitly threaded capabilities, alternatives, etc).

Parkinson and Bierman. Separation logic and abstraction. POPL 2005.

- Abstract predicates can represent a richer domain of abstract state (not limited to a finite number, can be parametric, etc).
- Typestates encode a simpler notion of abstraction, generally targets a more lightweight verification.

Paper includes additional Related Work.

## Summary

I. Encoding typestates using existential types in a substructural type-and-effect system.
2. Support both state-based and transition-based specifications of abstract state evolution.

Experimental Prototype Implementation: https://code.google.com/p/dead-parrot

- Future Work:

Sharing of resources through disconnected variables.

## Prototype

## JavaScript-based implementation, runs in browser.



## Summary

I. Encoding typestates using existential types in a substructural type-and-effect system.
2. Support both state-based and transition-based specifications of abstract state evolution.

Experimental Prototype Implementation: https://code.google.com/p/dead-parrot

- Future Work:

Sharing of resources through disconnected variables.

