Indeterminateness in Qualitative and Quantitative Reasoning

Daniel Zboril, Florian Metze
{zboril,metze}@phonetik.uni-muenchen.de
Institut für Phonetik und Sprachliche Kommunikation (IPSK)
Schellingstr. 3, 80799 München, Germany

Abstract

A central issue of expert systems is the man-machine-interface. One of its most ergonomic realizations is a natural language interface. Uncertainty and indeterminateness are main properties of natural language. While there are many formalisms dealing with uncertainty, there still is a strong need of those modeling indeterminateness. Therefore a formalism is presented which (a) formalizes the semantic interpretation of natural language and (b) provides an instrument for reasoning with information given in natural language. After a short outline of the problem the theoretical foundations are discussed, showing that it can be solved by a statistical formalism based on a trivalent logic. Finally the formalism is exemplified and compared to related theories.

1. Introduction

Asking for the percentage of people in California who made more than \$ 100.000 as their income in 1994 Jan de Leeuw [7] mentions the following difficulties:
(1) If a thief steals money, is that income?
(2) How about capital?
(3) We also did not specify what it means to be "in California".
(4) Do we mean one had to reside in California in 1994? In all of 1994? Or does it mean that we are interested in people who are in California right now?
The central issue in our context will be: What can be said about the truth value of the statement "x percent of the people in California had an income above \$ 100.000 in 1994" if all this information is lacking?

There are even simpler propositions leading to similar difficulties:

E1 Berlin is a big town.

E2 Probably, Pizarro did not find El-Dorado.

E1 may be indeterminate[1] depending on the context because "big" may be regarded as too vague. E2 may be indeterminate as well because of an unfulfilled presupposition of reference ("El-Dorado"). Furthermore "probably" expresses a degree of uncertainty[2].

To represent natural language semantically correct we need a formalism which can deal with the fact that neither the truth value "true" nor the truth value "false" can be assigned to a proposition because its content is indeterminate, i.e. at least three truth values are required. To formalize E2 we have to assign a degree of certainty to a proposition, i.e. we need a statistical formalism.

2. Models of uncertainty and indeterminateness

There are many models treating uncertainty in artificial intelligence on different levels. Only a few of them consider indeterminateness as well as uncertainty. Dubois and Prade give modeling imprecision as one of the reasons for developing their "Possibility Theory" [5]. As imprecision may be caused by vagueness and unfulfilled presupposition of reference it is very similar to indeterminateness. Shafer [9] models a situation where both the belief in a proposition's being true and its being false is low, i.e. there are three possibilities: One believes that the proposition is true, one believes that it is false, or one believes neither one nor the other. We will discuss these theories at the end of the paper.

Blau [2] was the first whose main object was a trivalent logic adequate to formalize natural language. Zboril [11] proposes a statistical formalism S_3 to integrate uncertainty into this logic. Based thereon we

[1] Indeterminateness denotes a state of ignorance concerning the meaning of a proposition.

[2] Uncertainty denotes a state of gradual ignorance of the correspondence of a proposition with the real world.

present a general framework of S_3 and apply S_3 to qualitative and quantitative reasoning.

3. The formalism S_j

To formulate S_j we adopt classical, i.e. two valued, logic and statistics (defined by the axioms of Kolmogoroff) as a meta-language. Wherever the context is clear we use natural language to increase readability. Thereby the following postulate is implied:

P1 A proposition has a certain truth value or not, tertium non datur.

Possible uncertainty concerning the assignment of truth values to propositions are not affected by P1.

As shown in the introduction we need a statistical formalism which is able to cope with more than two truth values, say n. Each proposition x_i may have the truth value j or not and therefore x_i may be a member of the set of all propositions with the truth value j, X_j, or not. As there may be uncertainty concerning the x_i's membership in X_j we introduce a membership function[3] $P(x_i \in X_j)$. For $P(x_i \in X_j)$ the following axioms are postulated. They are derived from the axioms of Kolmogoroff by applying them to the $x_i \in X_j$ relations regarding them as elements A_{ij} of a set $\mathcal{A}$ with its power set $2^{\mathcal{A}}$.

Be $\mathcal{M} = \{X_1, X_2, ..., X_n\}$, $\bigcap X_j = \emptyset$, $x_a \in \bigcup X_j$, $a, i, j, k, l, r, s \in \mathbf{N}$, and $r \geq s$.

A1 $P(x_a \in X_j) \geq 0$

A2.1 $P(x_a \in \bigcup X_j) = 1$

A2.2 $P((x_1 \vee x_2 ... \vee x_i ...) \in \bigcup_{j=r}^{s} X_j) = 1$, iff the x_i are mutually exclusive and collectively exhaustive with respect to $\bigcup_{j=r}^{s} X_j$

A3.1 $P((x_1 \vee x_2 ...) \in \bigcup_{j=r}^{s} X_j) = \sum_{i=1}^{\infty} P(x_i \in \bigcup_{j=r}^{s} X_j)$, iff $\bigwedge_{x_k, x_l}$ with $x_k, x_l \in \{x_1, x_2, ...\}$: $\neg((x_k \in \bigcup_{j=r}^{s} X_j) \wedge (x_l \in \bigcup_{j=r}^{s} X_j))$

A3.2 $P(x_a \in \bigcup_{j=r}^{s} X_j) = \sum_{j=r}^{s} P(x_a \in X_j)$

Though this notation of the axioms of S_j is not the simplest we choose it because it clarifies the structure of S_j. A1 assigns a real valued number to each relation of type "$x_i \in X_j$". By A2 $P(.)$ is normalized; the membership function of a membership which is certain equals one. Additivity of X_j and x_i considering membership relationships is given by A3.

According to the cardinality of $\mathcal{M}$ we refer to the presented formalism as $S_{|\mathcal{M}|}$.

The axioms given above are equivalent to the axioms of Kolmogoroff if $\mathcal{M} = \{X_{true}, X_{false}\}$, i.e. S_2 becomes equivalent to classical statistics when the appropriate definitions are added. The membership of a proposition x_a in X_j $(j = 1, 2)$ is given by the tuple $[P(x_a \in X_{true}), P(x_a \in X_{false})]$. As $\sum_j P(x_a \in X_j) = 1$ $P(x_a \in X_{true})$ is sufficient to describe the membership of x_a in X_j $(j = 1, 2)$.

The introduction implies that S_2 is not adequate to formalize the semantic interpretation of natural language. According to Blau, a proposition can be *true* (iff its meaning is clear and its content describes a state or process of the real world), *false* (iff its meaning is clear but its meaning does not correspond to a state or process of the real world), or *indeterminate* (iff its meaning is not clear and therefore nothing can be said about its correspondence to the real world) - quartum non datur. Therefore $\mathcal{M} = \{X_{true}, X_{false}, X_{indeterminate}\}$ in S_3. As $\sum_j P(x_a \in X_j) = 1$ the membership of x_a in X_j $(j = 1, 2, 3)$ can be fixed by a tuple.

When reasoning under certainty but still considering propositions which might be indeterminate S_3 is equivalent to the statement logic as developed by Blau.

4. Qualitative reasoning

The central issue of qualitative reasoning within the context of S_3 is to calculate the unknown membership function $P(x_a \in X_j)$ if the membership functions of some other propositions of which x_a can be composed by logical connectives are known. Nilsson [8] describes a formalism which seems to be able to deal with this problem. However, applying his linear approximation methods in numerical simulations we got results, which seemed not applicable in real world applications (e.g. given $P(x_a \in X_{true}) = 1$ and $P(x_b \in X_{true}) = 1$, $P((x_a \wedge x_b) \in X_{true})) = 0.56$ and $P((x_a \wedge x_b) \in \cup_{true,indeterminate} X_j) = 0.89$). We therefore propose an alternative method defining logical connectives in terms of S_3.

Introducing the probability tuples $\Pi\mathrm{T}(x_a) := [P^+, P^{+\circ}] := [P(x_a \in X_{true}), P((x_a \in X_{true}) \vee (x_a \in X_{indeterminate}))]$[4] we define for these ΠTs:

D1.1 $a \;:=\; [a, a]$

D1.2 $[a, b] + [c, d] \;:=\; [a + c, b + d]$

D1.3 $[a, b] - [c, d] \;:=\; [a - c, b - d]$

D1.4 $[a, b] \bullet [c, d] \;:=\; [a \cdot c, b \cdot d]$

D1.5 $[a, b] \circ [c, d] \;:=\; [a : c, b : d]$; $c, d \neq 0$

[3] The term "membership function", which is basic in Possibility Theory, is used in S_j as well. This should not lead to confusion. See also chapter 8.

[4] Looking at the following definitions one might easily verify that other notations of the probability tuples do not allow for their simple representation.

D1.1 permits to combine reasoning under S_2 with reasoning under S_3. Given the probability $P(x_a \in X_{true})$ under S_2 one might introduce it as $\Pi T(x_a)$ according to D1.1 in S_3 (and, mutatis mutandis, vice versa). D1.2 simplifies the representation of additivity and D1.4 allows to represent the membership of $(x_a \wedge x_b) \in X_j (j = 1, 2, 3)$ as $\Pi T(x_a \wedge x_b) = \Pi T(x_a) \bullet \Pi T(x_b)$ according to the following definition D2. D1.3 and D1.5 are just the inverse operations of D1.2 and D1.4 respectively.

We now define the logical connectives $-, \neg, \wedge, u, \vee, \rightarrow, \leftrightarrow$ in terms of S_3 given $\Pi T(x_a)$ and $\Pi T(x_b)$[5]:

D2.1 $NEG(\Pi T(x_a)) =$
$\Pi T(-x_a) =$
$[1 - P^{+\circ}, 1 - P^{+}]$

D2.2 $NOT(\Pi T(x_a)) =$
$\Pi T(\neg x_a) =$
$[1 - P^{+}, 1 - P^{+}]$

D2.3 $IND(\Pi T(x_a)) =$
$\Pi T(ux_a) =$
[0,1]

D2.4 $AND(\Pi T(x_a), \Pi T(x_b)) =$
$\Pi T(A \wedge B) =$
$\Pi T(x_a) \bullet \Pi T(x_b)$
(in case of A and B being independent)

D2.5 $ANDD_{a|b}(\Pi T(x_a), \Pi T(x_b)) =$
$\Pi T(x_a \mid x_b) \bullet \Pi T(x_b)$
(in case of A and B being dependent, where $\Pi T(x_a \mid x_b)$ denotes the conditional probability tuple of x_a given that x_b)

D2.6 $OR(\Pi T(x_a), \Pi T(x_b)) =$
$\Pi T(x_a \vee x_b) =$
$NEG(AND(NEG(\Pi T(x_a)),$
$NEG(\Pi T(x_b))))$

D2.7 $COND(\Pi T(x_a), \Pi T(x_b)) =$
$\Pi T(x_a \rightarrow x_b) =$
$OR(NOT(\Pi T(x_a)), \Pi T(x_b))$

D2.8 $BICOND(\Pi T(x_a), \Pi T(x_b)) =$
$\Pi T(x_a \leftrightarrow x_b) =$
$AND(COND(\Pi T(x_a), \Pi T(x_b)),$
$COND(\Pi T(x_b), \Pi T(x_a)))$

These definitions allow to calculate $\Pi T(x_a)$, if x_a can be constructed using other propositions x_i and logical connectives defined by D2, provided the $\Pi T(x_i)$ are known.

[5] D2.6-2.8 are based on D2.4 for reasons of simplicity.

5. Quantitative reasoning

Considering quantitative evidence under conditions of uncertainty and indeterminateness means computing with probability functions which might be discrete or continuous[6] and whose domain of definition reflects a quantitative feature (e.g. age, income etc.). The probability functions are of type $\Pi T(x_a(S) = x)$ (e.g. $\Pi T(Income(Mary) = x)$).

The membership of $x_i(S) = x$ in X_j $(j = 1, 2, 3)$ is given by

$$\Pi T(a \leq x_i(S) \leq b) = [\int_a^b f(x_i(S) = x)dx, \int_a^b g(x_i(S) = x)dx]$$

with $f(.)$ and $g(.)$ being probability functions according to the axioms. We may simplify this equation by introducing D3:

D3 $[\int_a^b f(x_i(S) = x)dx, \int_a^b g(x_i(S) = x)dx] := \int_a^b [f(x), g(x)]dx$

Not every tuple of type $[f(x), g(x)]$ defines a membership function for $x_i(S) = x$ in $X_j (j = 1, 2, 3)$. The tuple of functions must satisfy the following conditions:
(1) $\bigwedge_x : 0 \leq f(x) \leq g(x) \leq 1$
(2) Normalization: (a) $\int_{-\infty}^{\infty} f(x_i(S) = x)dx = 1$ or (b) $\int_{-\infty}^{\infty} g(x_i(S) = x)dx = 1$
By (2b) it becomes possible to express that $x_i(S) = x$ is indeterminate to a certain degree in general (even independent of the concrete value of x), e.g. $Income(Tree) = x$.

We now look at an example for combining evidence: Based on $\Pi T(x_a(S) = x \mid x_b(S))$ and $\Pi T(x_a(S) = x \mid x_c(S))$[7] we can calculate $\Pi T(x_a(S) = x \mid x_b(S), x_c(S))$ as follows, assuming $x_b(S)$ and $x_c(S)$ to be conditionally independent:

EQ1 $\Pi T(x_a(S) = x \mid x_b(S), x_c(S)) =$
$(\Pi T(x_a(S) = x \mid x_b(S))$
$\bullet \Pi T(x_a(S) = x \mid x_c(S))$
$\bullet \Pi T(x_b(S)) \bullet \Pi T(x_c(S)))$
$\circ (\Pi T(x_a(S) = x) \bullet \Pi T(x_b(S), x_c(S)))$

According to Cheeseman [4] $x_b(S)$ and $x_c(S)$ only have to be (conditionally) independent with respect to $x_a(S) = x$. However, nothing within the probabilistic formalism requires independence, i.e. it can deal with different degrees of dependence. In an extreme

[6] Using the concept of integral of Stieltje our notations are independent of the probability distributions' being discrete or continuous.

[7] $x_b(S)$ and $x_c(S)$ denote qualitative statements about X (e.g. Young(Mary))

case of dependence $\Pi T(x_a(S) = x \mid x_b(S), x_c(S)) = (\Pi T(x_a(S) = x \mid x_b(S))$.

6. Combining qualitative and quantitative reasoning

Qualitative and quantitative reasoning have to be combined in many different situations. Sometimes qualitative propositions are defined by quantitative ones (e.g. being young means that one's age is under 20 years). We then get terms like $\Pi T(x_a(S)) = \int_a^b \Pi T(x_b(S) = x)dx$. We might also calculate $\Pi T(x_a(S))$ given $\Pi T(x_a(S) \mid x_b(S) = x)$ and $\Pi T(x_b(S) = x)$ by

EQ2 $\Pi T(x_a(S)) =$
$\int_{min}^{max} \Pi T(x_a(S) \mid x_b(S) = x)$
$\bullet \Pi T(x_b(S) = x)dx$

Given $\Pi T(x_a(S))$ and $\Pi T(x_b(S) = x \mid x_a(S))$, we can calculate $\Pi T(x_b(S) = x)$ according to

EQ3 $\Pi T(x_b(S) = x) =$
$\Pi T(x_b(S) = x \mid x_a(S)) \bullet \Pi T(x_a(S))$
$+\Pi T(x_b(S) = x \mid \neg x_a(S)) \bullet \Pi T(\neg x_a(S))$

7. Examples

We will look at examples of qualitative reasoning first. In case of certainty about the truth values of a proposition x_a the probability tuples become $[1,1]$ for *true*, $[0,1]$ for *indeterminate* and $[0,0]$ for *false*. In this case reasoning under S_3 is equivalent to reasoning under the trivalent statement logic of Blau.

Asking for Mary's academic success $Successful(Mary)$ based on information about her age $Young(Mary)$, her mental abilities $Intelligent(Mary)$, and her attitude towards work $Lazy(Mary)$ this might be modeled as follows:
$\Pi T(Successful(Mary)) :=$
$AND(\Pi T(Young(Mary)),$
$OR(\Pi T(Intelligent(Mary)),$
$NEG(Lazy(Mary))))$
In our model Mary is successful if she is young and intelligent or young and not lazy. Given certainty we get:
$\Pi T(Successful(Mary)) = [1,1]$ if
$\Pi T(Young(Mary)) = [1,1]$ and
$\Pi T(Intelligent(Mary)) = [1,1]$, or
$\Pi T(Young(Mary)) = [1,1]$ and
$NEG(Lazy(Mary)) = [1,1]$.
This shows that for our model reasoning under S_3 given certainty corresponds to our intuition and is equivalent to reasoning under S_2 or ordinary first order logic.

We now regard our model under conditions of uncertainty and indeterminateness. Given for example:
$\Pi T(Young(Mary)) := [0.9, 0.95]$,
$\Pi T(Intelligent(Mary)) := [0.9, 0.95]$ and
$\Pi T(Lazy(Mary)) := [0.05, 0.1]$ one gets
$\Pi T(Successful(Mary)) = [0.89, 0.95]$.
If there is reason to believe that Mary tends not to be lazy (i.e. P^+ is very small) although *lazy* is a vague concept then $\Pi T(Lazy(Mary))$ might be given as:
$\Pi T(Lazy(Mary)) := [0.05, 0.5]$. We now find:
$\Pi T(Successful(Mary)) = [0.86, 0.95]$.
If we only know
$\Pi T(Young(Mary)) := [0.5, 0.95]$ and
$\Pi T(Lazy(Mary)) := [0.05, 0.1]$ we find
$\Pi T(Successful(Mary)) = [0.50, 0.95]$.

Our model is very sensitive to the indeterminateness of $Young(Mary))$ while indeterminateness concerning either Mary's I.Q. or diligence would not necessarily cause indeterminateness of $Successful(Mary)$. This is implied by the underlying logical structure of the model.

If $\Pi T(Young(Mary))$ is not directly known but $\Pi T(Young(Mary) \mid Age(Mary) = x)$ and $\Pi T(Age(Mary) = x)$[8] are given, $\Pi T(Young(Mary))$ can be calculated according to **EQ2**.

What can be said about
$\Pi T(Age(Mary) = x \mid \neg Young(Mary))$ given
$\Pi T(Age(Mary) = x \mid Young(Mary))$[9],
$\Pi T(Age(Mary) = x)$, and $\Pi T(Young(Mary))$?
$\Pi T(Age(Mary) = x \mid \neg Young(Mary)) =$
$(\Pi T(Age(Mary) = x) -$
$(NOT(NEG(\Pi T(Age(Mary) = x \mid Young(Mary))))$
$\bullet\ NOT(NEG(\Pi T(Young(Mary))))))$
$\circ\ (NOT(\Pi T(Young(Mary)))$

$\Pi T(Young(Mary))$ can be calculated like we did in the previous example. There is no indeterminateness in the result as the truth value of $Young(Mary)$ is given by condition and $Age(Mary) = x$ is indeterminate neither due to vagueness nor due to unfulfilled presupposition of reference.

The previous examples on quantitative reasoning are straightforward generalizations of Cheeseman's [4] procedures for reasoning with this kind of information under S_2.

Asked for Mary's income an answer like *Mary earns a good salary* would not be very satisfying. Therefore S_3 must provide an optimal single value based on the information given, i.e. $\Pi T(Income(Mary) = x \mid GoodSalary(Mary))$. This value can be calcu-

[8] $\Pi T(Age(Mary) = x)$ equals $\Pi T(Age(S) = x)$ (with $\Pi T(Age(Mary) = x)$ being the age probability function of human beings) if nothing is known about Mary which might have an influence on the estimation of her age.

[9] For formal reasons a general context C might be introduced, e.g. $\Pi T(Age(Mary) = x \mid Young(Mary), C)$, to model all general facts known about Mary, like $Student(Mary)$. This is neglected here for reasons of simplicity.

lated by "center of mass" or "least square fit" methods depending on the context. The problem is that $P^{+}(Income(Mary) = x \mid GoodSalary(Mary))$ will give us another value than $P^{+\circ}(Income(Mary) = x \mid GoodSalary(Mary))$. The question is, on which of the two values our answer is to be based. As we want to know Mary's true income we choose the value based on $P^{+}(Income(Mary) = x \mid GoodSalary(Mary))$. If we get $Income(Mary) = y$ as a result we can give its indeterminateness as $P^{+\circ}(Income(Mary) = y \mid GoodSalary(Mary)) - P^{+}(Income(Mary) = y \mid GoodSalary(Mary))$.

Applied in an expert system to model man-machine interaction S_3 provides three types of reactions for the system after a process of reasoning. These are (considering the inquiry $?Student(Mary)$ as an example with $\Pi T(Student(Mary))$):
(1) It can confirm the proposition *Student(Mary)* (e.g. if $P^{+} > P^{+\circ} - P^{+}$ and $P^{+} > 1 - P^{+}$).
(2) It can explicitly disconfirm the proposition (e.g. if $1 - P^{+} > P^{+\circ} - P^{+}$ and $1 - P^{+} > P^{+}$).
(3) It can reject the question or ask for some more information (in all other cases).
The possibility to differentiate between the second and the third option is given by S_3 and is not available in S_2.

8. Related formalisms

Based on the paradigm of Grosof [6] S_3 can be characterized as a Type-1-ue[10] probabilistic theory according to its axioms. However, conditional probabilities are introduced by D2. S_3 thereby becomes of Type-1-ce. By introducing conditional independence (e.g. in D2 or EQ1) as obligatory (instead of optional as we did) S_3 becomes of non-Type-1.

According to this classification Shafer's theory of evidence is of Type-2 because of Dempster's Rule. The belief functions are special cases of upper-lower distributions of Type-1-i theories. S_3 is not given in terms of inequality. Therefore $P(x_i \in X_{indeterminate})$ does not represent the size of an interval containing the "real" probability of a proposition x_i. Furthermore, this implies that $P(x_i \in X_{indeterminate})$ cannot be given by the possibility and necessity of x_i.

Having thus classified Shafer's theory of evidence and S_3 formally, we now compare them by presenting an example: x_a=(*Is the Antarctic inhabited?*). According to Shafer we get $\Omega = \{x_a, \bar{x_a}\}$ and $2^{\Omega} = \{\{x_a, \bar{x_a}\}, \{x_a\}, \{\bar{x_a}\}, \emptyset\}$. Three situations arise:

(1) If the meaning of x_a is clear and we have knowledge about x_a, the mass $m(\Omega)$ will be close to 0, i.e. nearly all of the probability mass is bound either to x_a or to $\bar{x_a}$.

(2) If we do not know very much about x_a while its meaning is clear, $m(x_a)$ and $m(\bar{x_a})$ is near 0 and therefore $m(\Omega)$ tends to 1. This means that (caused by the underlying two valued logic) we know that x_a or $\bar{x_a}$ must be true, but we cannot decide which one.

(3) In case of x_a's meaning being indeterminate (e.g. if the meaning of "inhabited" is too vague: Does it mean, that one single person has to stay there for a whole year? For more than a year? Or can it be different persons at different times? ...) the probability mass will be distributed as in the previous case, i.e. we can not differentiate between an indeterminate proposition and a proposition which may be well understood but about whose correspondence to the real world nothing can be said.

While S_3 can deal with indeterminateness adequately the second situation can not be modeled because S_3 (as far as developed here) is of Type-1-e. S_3 can easily be extended to a Type-1-i (or a Type-2) formalism by introducing probability intervals representing uncertainty concerning the real value of $P(x_i \in X_j)$. This could be done by basing the approach of Weichselberger [10] on S_3 instead of S_2.

Possibility Theory is said to be "a development of Fuzzy Set theory, for representing vagueness in some linguistic terms"[11] [1]. Clarifying the differences between Possibility Theory and S_3 we will not look at those already discussed within the context of S_2 (e.g. [3]), such as "the "Fuzzy" Combination Rule" [6].

Dubois and Prade give an example of modeling an unfulfilled presupposition of reference: They regard $Age(Car(S))$, the age of the car of a person S. D denotes the set of all possible values of $Age(Car(S))$. In addition, they introduce the residual element e. Then $\Omega = \{D \cup \{e\}\}$. If now the presupposition of reference is unfulfilled, e.g. if S has no car, this is expressed by assigning the possibility 0 to D and the possibility 1 to $\{e\}$. We can model this situation analogous under S_3 in terms of probability theory. If $P(x_a(S) = x \in X_{indeterminate})$ depends on x, reasoning under Possibility Theory is quite similar to reasoning under Shafer's theory, i.e. indeterminateness as defined in the introduction cannot be modeled. For some vague terms Possibility Theory gives a definition in form of a possibility distribution. If e.g. $Tall(S)$ is to be repres-

[10] Type-1 characterizes theories based on the axioms of Kolmogoroff. They might include conditional probabilities (c) or not (u) and be given in terms of equalities (e) or inequalities (i). If theories are based on Type-1-ci and make further assumptions or include other rules, they are said to be of Type-2 (e.g. Dempster-Shafer theory).

[11] "Vagueness" is used synonymously to our notion of "indeterminateness" rather than our notion of "vagueness".

ented the possibility distribution might be given over all possible values of $Size(S)$. This can be done in S_3 in a similar way, e.g. by giving the probability function of $\Pi\mathrm{T}(Possible(Tall(S)) \mid Size(S))$.

Next to formal differences such as additivity and normalization (which are mutatis mutandis equivalent to those between Possibility Theory and classical statistics) and those concerning modeling real world problems, there are those concerning the terminology, i.e. semantical differences. In S_3 the terms "membership function of x_a's being in the set of propositions with the truth value j" and "probability function of x_a's being assigned to the truth value j" are equivalent. This equivalence is justified by the axiomatic basis of S_3 and therefore not given in Possibility Theory.

9. Conclusion

With S_3 a formalism is presented which is adequate to represent uncertainty and indeterminateness of natural language. For representing uncertainty on a higher level, i.e. uncertainty when estimating the degree of uncertainty (e.g. in form of probabilities) S_3 can easily be extended to a theory of Type-1-i or of Type-2. S_3 is based on the axioms of Kolmogoroff and in all respects consistent with ordinary first order logic, which might be derived from S_3: We reduce $\mathcal{M} = \{X_{true}, X_{false}, X_{indeterminate}\}$ to $\mathcal{M} = \{X_{true}, X_{false2}\}$ with $X_{false2} = \cup_{false,indeterminate} X_j$. Having thus derived S_2 we get ordinary first order logic by introducing the restriction $P(.) \in \{0, 1\}$.

Future work should be focused on the implementation of real-world applications. Within this framework a decision making tool has to be designed to refine the basic rules governing the interpretation of the probability tuples. S_3 provides a new approach to the problem of the semantic interpretation of natural language for developing man-machine-interfaces.

The relations between S_3 and other formalisms for reasoning under conditions of uncertainty and indeterminateness have to be evaluated in detail to guarantee a useful exchange of information between different layers in expert systems based on different ones of these formalisms. At present none of the formalisms can be used to model the whole area of indeterminateness and uncertainty, but with S_3 a powerful basis for further development is provided.

10. Acknowledgments

R. Kossmann, J. Inthorn, and colleagues gave helpful comments on earlier drafts of this paper.

11. References

[1] Bhatnagar, R.K. and Kanal, L.N. (1986): Handling Uncertain Information: A Review of Numeric and Non-numeric Methods; in: Kanal, L.N. and Lemmer, J.F. (eds.): Uncertainty in Artificial Intelligence; Elsevier, North Holland.

[2] Blau, U. (1978): Die dreiwertige Logik der Sprache; de Gruyter, Berlin.

[3] Cheeseman, P. (1985): In defense of probability; Proceedings of IJCAI.

[4] Cheeseman, P. (1986): Probabilistic versus Fuzzy Reasoning; in: Kanal, L.N. and Lemmer, J.F. (eds.): Uncertainty in Artificial Intelligence; Elsevier, North Holland.

[5] Dubois, D. and Prade, H. (1988): Possibility Theory; Plenum, New York.

[6] Grosof, B.N. (1986): An Inequality Paradigm for Probabilistic Knowledge; in: Kanal, L.N. and Lemmer, J.F. (eds): Uncertainty in Artificial Intelligence; Elsevier, North Holland.

[7] Leeuw, J. de (1996): Statistics. The Study of Variability in Variation; http://www.stat.ucla.edu/textbook/introduction/models/.

[8] Nilsson, N.J. (1986): Probabilistic Logic; Artificial Intelligence, 28, North-Holland.

[9] Shafer, G. (1976): A mathematical theory of evidence; Princeton University, Princeton.

[10] Weichselberger, K. and Pöhlmann, S. (1990): A Methodology for Uncertainty in Knowledge-Based Systems; Springer, Berlin.

[11] Zboril, D. (1996): Modelling Indeterminateness or how to integrate probability and logic; COMPSTAT 96 - forthcoming, UPC, Barcelona.