Incentive compatibility (IC)

Fundamental concept in mechanism design
Buyers maximize their utilities by bidding truthfully

Many real-world mechanisms are not IC:
- Discriminatory auctions
  - Multi-unit variant of first-price auction
  - Used to sell US treasury bills and UK electricity
- Generalized second price auction
  - Used for sponsored search
- First-price auction
  - Display ad exchanges may be transitioning to FP
- Most fielded combinatorial auctions
  - For example, sourcing auctions

Why not?
- Might be expensive for buyers to compute true values
- Rules are often easier to explain
- Bids used to tune future auction
- Auction might leak the bidders’ private values
- Bidders might not be risk neutral

Approximate IC

Mechanism is $\gamma$-IC when for each bidder $i$:
- If everyone except bidder $i$ is truthful, she can only increase utility by $\gamma$ if she bids strategically

Defined either:
1. In expectation over others’ values (ex-ante)
2. In expectation over all values (ex-ante)

Studied extensively:

Literature on $\gamma$-IC assumes distribution is known in advance

Where does this knowledge come from?

We relax this assumption:
Assume only samples from type distribution

Estimating approximate IC

Overriding goal:
Estimate IC approximation factor ($\gamma$) using samples

Our estimate:
Maximum utility agent can gain by misreporting her type, on average over samples, when true & reported types from finite subset of type space

Estimate used in mechanism design via deep learning:
Add constraint requiring this estimate be small
[Dütting et al., ‘17, Feng et al., ‘18, Golowich et al., ‘18]

Challenge:
Might miss true & reported types with large utility gains

Crucial questions:
1. Which finite subset?
2. What’s the estimation error?

Which finite subset?
1. Uniform grid: Focus of this poster
   - Easy to construct
   - Works if distribution is “nice”
2. Learning theoretic cover (standard from ML theory)
   - Can be hard to construct
   - Always works

Uniform grid: Main challenge

Utility functions are volatile

Coarse discretization can lead to poor utility estimates

Uniform grid: Guarantees

When is the distribution “nice” enough to use a grid?

Dispersion [Balcan, Dick, and Vitercik, ‘18]:
Functions $u_1, \ldots, u_N$ are $(w, k)$-dispersed if:
Every $w$-ball contains discontinuities of $\leq k$ functions

Theorem (informal):
If utility functions induced by $N$ samples are:
1. $(w, k)$-dispersed
2. Piecewise $L$-Lipschitz
   - Can use $w$-grid as finite subset

Estimation error: $\hat{\gamma} \left( \frac{Lw + k}{N} + \frac{d}{\sqrt{N}} \right)$

$d = \text{standard ML measure of utility functions' complexity}$

Error

When $w = O\left(\frac{1}{\sqrt{N}}\right), k = O\left(\sqrt{N}\right)$:

We prove these $(w, k)$ values hold when distribution is nice

Applications

$[0, \kappa] = \text{range of type distribution's density function}$

First-price auction
Estimation error $= \hat{\gamma} \left( \frac{(\#\text{bidders}) + \kappa^{-1}}{\sqrt{(\#\text{samples})}} \right)$

Also analyze combinatorial first-price auctions

Generalized second-price auction
Estimation error $= \hat{\gamma} \left( \frac{(\#\text{bidders})^{2/3} + \kappa^{-1}}{\sqrt{(\#\text{samples})}} \right)$

Discriminatory and uniform price auctions
Generalization of first-price auction to multi-unit settings
Estimation error $= \hat{\gamma} \left( \frac{(\#\text{bidders})(\#\text{units})^{2/3} + \kappa^{-1}}{\sqrt{(\#\text{samples})}} \right)$