



Consider the following grammar:

$$\begin{aligned} S &\rightarrow aSa \mid A \mid C \\ A &\rightarrow bBb \mid bCb \mid E \\ B &\rightarrow bBb \mid \lambda \\ C &\rightarrow aC \mid bC \\ D &\rightarrow aD \mid \lambda \\ E &\rightarrow bb \mid bEb \end{aligned}$$

**Problem 1**

Simplify this grammar: remove all useless variables,  $\lambda$ -productions, and unit-productions.

The simplification gives the following result:

$$\begin{aligned} S &\rightarrow aSa \mid bb \mid bBb \mid bEb \\ B &\rightarrow bb \mid bBb \\ E &\rightarrow bb \mid bEb \end{aligned}$$

Since  $B$  and  $E$  give rise to the same strings, we may remove either of them:

$$\begin{aligned} S &\rightarrow aSa \mid bb \mid bBb \\ B &\rightarrow bb \mid bBb \end{aligned}$$

This grammar generates the language  $\{a^n b^{2m} a^n : n \geq 0, m \geq 1\}$ .

**Problem 2**

Give an equivalent grammar in Chomsky normal form.

$$\begin{aligned} S &\rightarrow XC \mid YY \mid YD \\ C &\rightarrow SX \\ D &\rightarrow BY \\ X &\rightarrow a \\ Y &\rightarrow b \end{aligned}$$

**Problem 3**

Give an equivalent grammar in Greibach normal form.

$$\begin{aligned} S &\rightarrow aSX \mid bY \mid bBY \\ B &\rightarrow bY \mid bBY \\ X &\rightarrow a \\ Y &\rightarrow b \end{aligned}$$