Automata Theory: Solutions 7

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homeworks              X     X
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               grades
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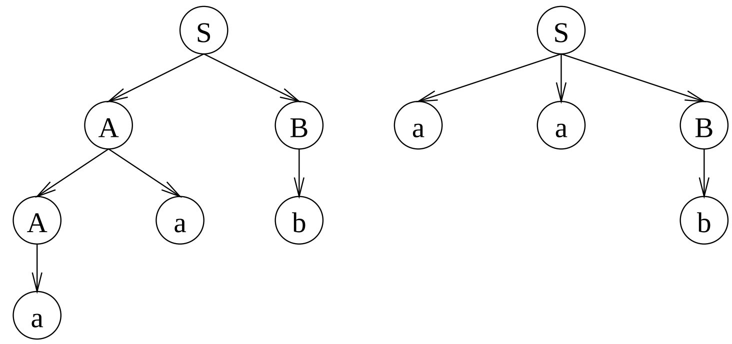
Problem 1
Consider the following grammar:

$S \to AB \mid aaB$
$A \to a \mid Aa$
$B \to b$

(a) Show that this grammar is ambiguous.

We may derive aab using either $S \to AB$ or $S \to aaB$, which give rise to different trees:



(b) Give a regular expression that describes the same language.

The language comprises all words that consist of one or more a's followed by exactly one b; thus, the corresponding regular expression is $\boldsymbol{a^+b}$.

(c) Construct an unambiguous grammar that describes the same language.

We can remove the production $S \to aaB$ to eliminate the ambiguity, which leads to the following grammar:

$S \to AB$
$A \to a \mid Aa$
$B \to b$

We can further simplify this grammar, by eliminating the variables A and B:

$S \to aS \mid ab$

Problem 2
For each of the following two grammars, construct an equivalent grammar that has no λ-productions and no unit-productions.

(a) $S \to A \mid B$
$A \to a \mid aA$
$B \to b \mid bB$

Simplified grammar:

$S \to a \mid aA \mid b \mid bB$
$A \to a \mid aA$
$B \to b \mid bB$

(b) $S \to aAb \mid bAa \mid aSb \mid bSa$
$A \to aAa \mid \lambda$

Simplified grammar:

$S \to aAb \mid bAa \mid aSb \mid bSa \mid ab \mid ba$
$A \to aAa \mid aa$