

Automata Theory: Solutions 6

									X
									X
			X				X		X
			X		X		X		X
			X		X	X	X		X
			X	X	X	X	X		X
			X	X	X	X	X	X	X
			X	X	X	X	X	X	X
		X	X	X	X	X	X	X	X
	X	X	X	X	X	X	X	X	X
	X	X	X	X	X	X	X	X	X
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	2	3	4	5	6	7	8	9	10
	grades								

Problem 1

Give a context-free grammar for each of the following languages:

- $\{a^n b^{2n} : n \geq 0\}$
- $\{a^n b^m c^{n+m} : n, m \geq 0\}$
- $\{a^n b^m : 2n \leq m \leq 3n\}$

- $$\begin{array}{l} \text{(a) } S \rightarrow aSbb \mid \lambda \\ \text{(b) } S \rightarrow aSc \mid A \\ \quad A \rightarrow bAc \mid \lambda \\ \text{(c) } S \rightarrow aSbb \mid aSbbb \mid \lambda \end{array}$$

Problem 2

Consider the following grammar:

$$\begin{array}{l} S \rightarrow SA \mid A \\ A \rightarrow aAb \mid \lambda \end{array}$$

Show a left-most derivation, right-most derivation, and derivation tree for the string $abaabb$.

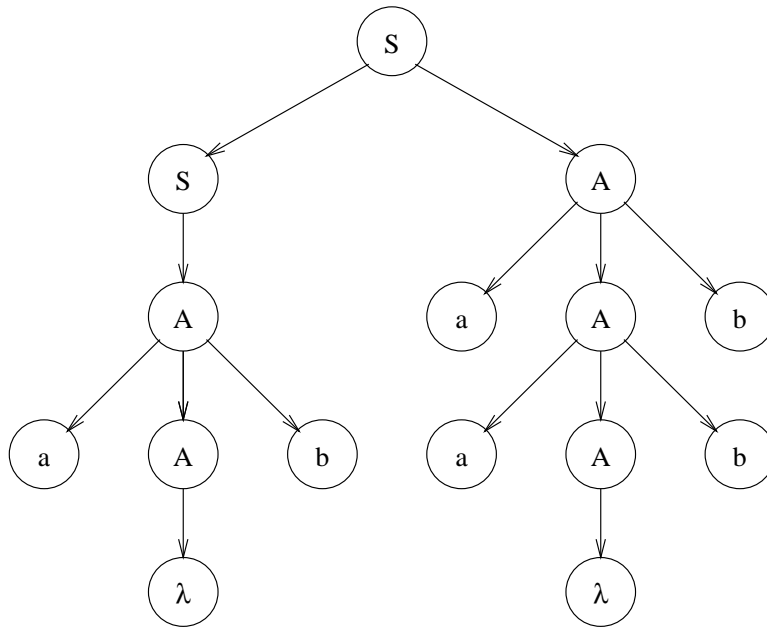
Left-most derivation:

$$S \Rightarrow SA \Rightarrow AA \Rightarrow aAbA \Rightarrow abA \Rightarrow abaAb \Rightarrow abaaAbb \Rightarrow abaabb$$

Right-most derivation:

$$S \Rightarrow SA \Rightarrow SaAb \Rightarrow SaaAbb \Rightarrow Saabb \Rightarrow Aaabb \Rightarrow aAbaabb \Rightarrow abaabb$$

Derivation tree:



Problem 3

Find *simple* grammars (a.k.a. s-grammars) for the following languages:

(a) $\{a^n b : n = 0 \text{ or } n \geq 3\}$

(b) $\{a^n b^n : n \geq 1\}$

(a) $S \rightarrow aA \mid b$

$A \rightarrow aB$

$B \rightarrow aC$

$C \rightarrow aC \mid b$

(b) $S \rightarrow aA$

$A \rightarrow aAB \mid b$

$B \rightarrow b$