Automata Theory: Solutions 7

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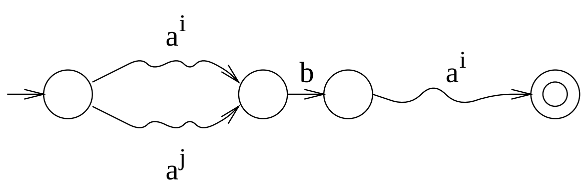
                                    X
                                    X
                                    X
                                    X
      number of                     X  X
      homeworks                     X  X
                                    X  X
                                 X  X  X
                                 X  X  X
                  X     X        X  X  X
                  X  X  X     X  X  X  X
            X     X  X  X  X  X  X  X  X
            -----------------------------
            1  2  3  4  5  6  7  8  9  10
                       grades
```

Problem 1
Argue that the language $\{a^n b^k a^n : \ k, n \geq 0\}$ is not regular.

We suppose that the language is regular and derive a contradiction. If it is regular, there is some m-state DFA that accepts it, where m is an unknown fixed number. Suppose that we begin from the initial state of the DFA and trace the path for each of the following strings: $a^0, a^1, a^2, ..., a^m$. Since the number of strings is $m+1$, two of them must lead to the same state; we denote these two strings a^i and a^j (see the picture). Since the automaton accepts $a^i b a^i$, this string leads from the initial state to a final state, as shown in the picture. Then, $a^j b a^i$ leads to the same final state, contradicting the fact that $a^j b a^i$ is *not* in the language.



Problem 2
Give a context-free grammar for each of the following languages:

(a) $\{a^n b^k a^n : \ k, n \geq 0\}$
(b) $\{a^k b^m a^n : \ m = 2 \cdot k + 2 \cdot n\}$
(c) $\{a^k b^m a^n : \ k + m \geq n\}$

(a) $S \to aSa \mid A$
$A \to bA \mid \lambda$

(b) $S \to AB$
$A \to aAbb \mid \lambda$
$B \to bbBa \mid \lambda$

(c) $S \to aSa \mid aS \mid A$
$A \to bAa \mid bA \mid \lambda$