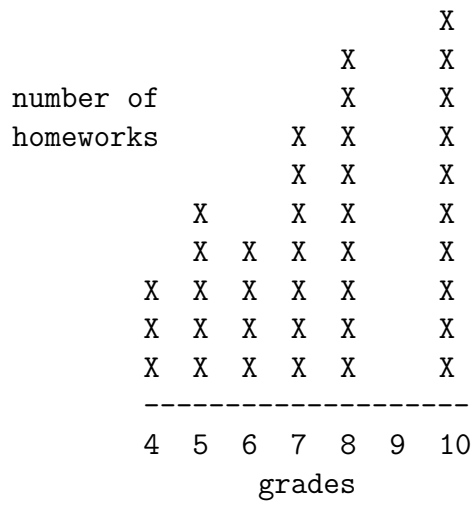
Automata Theory: Solutions 6



Problem 1
Show that, if L_1 is a regular language on $\Sigma = \{a, b\}$, the following language is also regular:

$$L_2 = \{w : \ w \in L_1 \text{ and the length of } w \text{ is odd}\}.$$

Consider the language L_3 defined as $((a + b) \cdot (a + b))^*$; since we describe L_3 by a regular expression, it is a regular language. This language includes all strings of even length, and we can express L_2 as follows:

$$L_2 = L_1 - L_3.$$

Thus, L_2 is the difference of two regular languages, which implies that it is also regular.

Problem 2
Describe a method for determining whether a given regular language L includes any string w such that w^R is also in L.

The key observation is that L includes such a string if and only if $L \cap L^R$ is nonempty. Given DFAs for L_1 and L_2, we can construct a DFA for $L \cap L^R$ and check whether the corresponding language is empty. If the resulting automaton does not accept any strings, then $L \cap L^R$ is empty, which means that L does not include any string whose reverse is also in L. If the automaton accepts some strings, then L includes some string w such that w^R is in L.