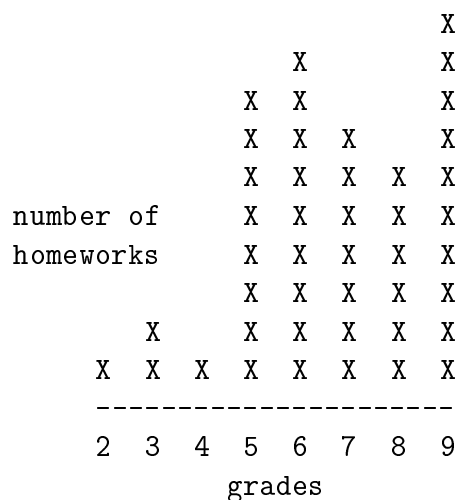


# Automata Theory: Solutions 3



The histogram shows the distribution of grades.

## Problem 1

Consider two languages on the alphabet  $\Sigma = \{a, b\}$ :

$$L_1 = \{a^n : n \geq 1\}$$

$$L_2 = \{ab^n : n \geq 0\}$$

Describe the following languages:

- $L_3 = L_1^* = \{a^n : n \geq 0\}$   
(the empty string and all strings that have no  $b$ 's)
- $L_4 = L_1^+ = \{a^n : n \geq 1\}$   
(all nonempty strings that have no  $b$ 's)
- $L_5 = \overline{L_1} = \{\lambda\} \cup \{w : w \text{ includes at least one } b\}$   
(the empty string and all strings that have at least one  $b$ )
- $L_6 = L_2^* = \{\lambda\} \cup \{aw : w \text{ is any string on } \Sigma\}$   
(the empty string and all strings that begin with  $a$ )
- $L_7 = L_1 \cap L_2 = \{a\}$
- $L_8 = L_1 L_2 = \{a^m b^n : m \geq 2 \text{ and } n \geq 0\}$

## Problem 2

Is there a language  $L$  on the alphabet  $\Sigma = \{a, b\}$  for which  $(\overline{L})^+ = \overline{L^+}$ ?

We consider the “empty” language,  $L = \emptyset$ , and show that it satisfies the equality.

The complement and positive closure of the empty language are as follows:

$$\begin{array}{ll} \overline{L} = \Sigma^* & L^+ = \emptyset \\ (\overline{L})^+ = \Sigma^* & \overline{L^+} = \Sigma^* \end{array}$$

We thus conclude that  $(\overline{L})^+ = \overline{L^+} = \Sigma^*$ .