Automata Theory: Solutions 3

```
                                        X
                            X           X
                        X   X           X
                        X   X   X       X
                        X   X   X   X   X
number of               X   X   X   X   X
homeworks               X   X   X   X   X
                        X   X   X   X   X
                X       X   X   X   X   X
            X   X   X   X   X   X   X   X
            -----------------------------
            2   3   4   5   6   7   8   9
                       grades
```

The histogram shows the distribution of grades.

Problem 1
Consider two languages on the alphabet $\Sigma = \{a, b\}$:

$L_1 = \{a^n : \ n \geq 1\}$
$L_2 = \{ab^n : \ n \geq 0\}$

Describe the following languages:

- $L_3 = L_1^* = \{a^n : \ n \geq 0\}$
  (the empty string and all strings that have no b's)
- $L_4 = L_1^+ = \{a^n : \ n \geq 1\}$
  (all nonempty strings that have no b's)
- $L_5 = \overline{L_1} = \{\lambda\} \cup \{w : \ w \text{ includes at least one } b\}$
  (the empty string and all strings that have at least one b)
- $L_6 = L_2^* = \{\lambda\} \cup \{aw : \ w \text{ is any string on } \Sigma\}$
  (the empty string and all strings that begin with a)
- $L_7 = L_1 \cap L_2 = \{a\}$
- $L_8 = L_1 L_2 = \{a^m b^n : \ m \geq 2 \text{ and } n \geq 0\}$

Problem 2
Is there a language L on the alphabet $\Sigma = \{a, b\}$ for which $(\overline{L})^+ = \overline{L^+}$?

We consider the "empty" language, $L = \emptyset$, and show that it satisfies the equality. The complement and positive closure of the empty language are as follows:

$$\overline{L} = \Sigma^* \qquad\qquad L^+ = \emptyset$$
$$(\overline{L})^+ = \Sigma^* \qquad\qquad \overline{L^+} = \Sigma^*$$

We thus conclude that $(\overline{L})^+ = \overline{L^+} = \Sigma^*$.