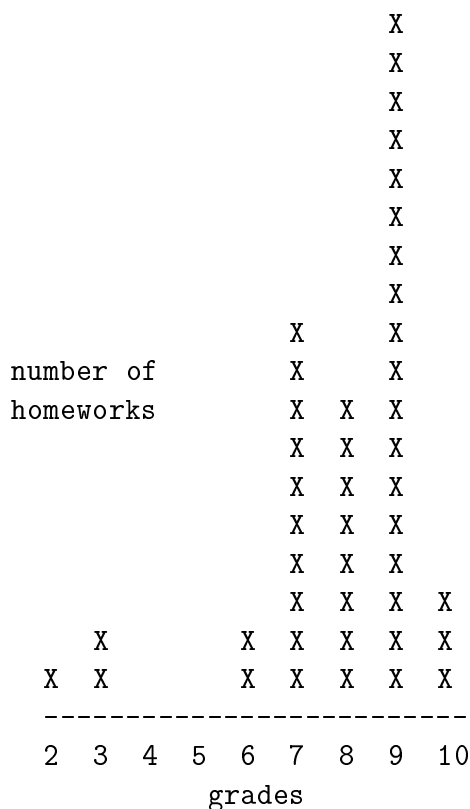


Automata Theory: Solutions 2



The histogram shows the distribution of grades.

Problem 1

Consider the following sets of integer numbers:

$$S_1 = \{1, 2, 3\}$$

$$S_2 = \{i : i \text{ is odd}\}$$

$$S_3 = \{i : i \text{ is divisible by } 3\}$$

For each set below, specify its elements:

$$S_4 = 2^{S_1} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

$$S_5 = 2^{S_1} \cap 2^{S_2} = \{\emptyset, \{1\}, \{3\}, \{1, 3\}\}$$

$$S_6 = S_3 - S_2 = \{i : i \text{ is divisible by } 6\}$$

The sets S_4 and S_5 are finite, whereas S_6 is infinite.

Problem 2

Prove that, if $S_1 \subseteq S_2$, then $2^{S_1} \subseteq 2^{S_2}$.

By the definition of subsets, we need to show that every element s of 2^{S_1} is also an element of 2^{S_2} . The proof is as follows:

$$\begin{aligned}
 s \in 2^{S_1} &\Leftrightarrow s \subseteq S_1 \quad (\text{by definition of powerset}) \\
 &\Rightarrow s \subseteq S_2 \quad (\text{because } S_1 \subseteq S_2) \\
 &\Leftrightarrow s \in 2^{S_2} \quad (\text{by definition of powerset})
 \end{aligned}$$

Problem 3

Prove the following equality by induction:

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n \cdot (n+1) \cdot (2 \cdot n + 1)}{6}.$$

Clearly, it holds for $n = 1$; we now show that, if it holds for n , then it also holds for $n + 1$:

$$\begin{aligned} 1^2 + 2^2 + \dots + n^2 + (n+1)^2 &= \frac{n \cdot (n+1) \cdot (2 \cdot n + 1)}{6} + (n+1)^2 \\ &= \frac{n \cdot (n+1) \cdot (2 \cdot n + 1)}{6} + \frac{6 \cdot (n+1)^2}{6} \\ &= \frac{(n+1)}{6} \cdot (n \cdot (2 \cdot n + 1) + 6 \cdot (n+1)) \\ &= \frac{(n+1)}{6} \cdot (2 \cdot n^2 + 7 \cdot n + 6) \\ &= \frac{(n+1)}{6} \cdot (n+2)(2 \cdot n + 3) \\ &= \frac{(n+1) \cdot ((n+1) + 1) \cdot (2 \cdot (n+1) + 1)}{6} \end{aligned}$$