Automata Theory: Solutions 8

```
                        X
                        X
                        X
                        X
                        X
                        X
                        X
                        X
                        X
                        X
                        X
                        X
                        X
                        X
                        X
number of               X
homeworks               X
                        X
                     X  X
                     X  X
                  X  X  X
                  X  X  X
   X     X     X  X  X  X
   -----------------------
   3  4  5  6  7  8  9  10
           grades
```

Consider the following grammar:

$$
\begin{aligned}
&S \rightarrow aSa \mid A \mid C \\
&A \rightarrow bBb \mid bCb \mid E \\
&B \rightarrow bBb \mid \lambda \\
&C \rightarrow aC \mid bC \\
&D \rightarrow aD \mid \lambda \\
&E \rightarrow bb \mid bEb
\end{aligned}
$$

Problem 1
Simplify this grammar: remove all useless variables, λ-productions, and unit-productions.

The simplification gives the following result:

$$S \to aSa \mid bb \mid bBb \mid bEb$$
$$B \to bb \mid bBb$$
$$E \to bb \mid bEb$$

Since B and E give rise to the same strings, we can remove either of them:

$$S \to aSa \mid bb \mid bBb$$
$$B \to bb \mid bBb$$

This grammar generates the language $\{a^n b^{2m} a^n : \; n \geq 0, m \geq 1\}$.

Problem 2
Give an equivalent grammar in Chomsky normal form.

$$S \to XC \mid YY \mid YD$$
$$C \to SX$$
$$D \to BY$$
$$X \to a$$
$$Y \to b$$

Problem 3
Give an equivalent grammar in Greibach normal form.

$$S \to aSX \mid bY \mid bBY$$
$$B \to bY \mid bBY$$
$$X \to a$$
$$Y \to b$$