Automata Theory: Solutions 8

```
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
                X
number of       X
homeworks       X
                X
              X X
              X X
            X X X
            X X X
  X   X   X X X X
  ----------------
  3 4 5 6 7 8 9 10
       grades
```

Consider the following grammar:

$S \rightarrow aSa \mid A \mid C$
$A \rightarrow bBb \mid bCb \mid E$
$B \rightarrow bBb \mid \lambda$
$C \rightarrow aC \mid bC$
$D \rightarrow aD \mid \lambda$
$E \rightarrow bb \mid bEb$

Problem 1
Simplify this grammar: remove all useless variables, λ-productions, and unit-productions.

The simplification gives the following result:

$$S \to aSa \mid bb \mid bBb \mid bEb$$
$$B \to bb \mid bBb$$
$$E \to bb \mid bEb$$

Since B and E give rise to the same strings, we can remove either of them:

$$S \to aSa \mid bb \mid bBb$$
$$B \to bb \mid bBb$$

This grammar generates the language $\{a^n b^{2m} a^n : \ n \geq 0, m \geq 1\}$.

Problem 2
Give an equivalent grammar in Chomsky normal form.

$$S \to XC \mid YY \mid YD$$
$$C \to SX$$
$$D \to BY$$
$$X \to a$$
$$Y \to b$$

Problem 3
Give an equivalent grammar in Greibach normal form.

$$S \to aSX \mid bY \mid bBY$$
$$B \to bY \mid bBY$$
$$X \to a$$
$$Y \to b$$