Automata Theory: Solutions 7

Problem 1

Give a context-free grammar for each of the following languages:

(a)
$$\{a^n b^{2n}: n \ge 0\}$$

(b)
$$\{a^n b^m c^{n+m}: n, m \ge 0\}$$

(c)
$$\{a^n b^m c^k : n+k \le m \le 2n+2k\}$$

(a)
$$S \to aSbb \mid \lambda$$

(b)
$$S \to aSc \mid A$$

 $A \to bAc \mid \lambda$

(c)
$$S \to AB$$

 $A \to aAb \mid aAbb \mid \lambda$
 $B \to bBc \mid bbBc \mid \lambda$

Problem 2

Consider the following grammar:

$$S \to SA \mid A$$
$$A \to aAb \mid \lambda$$

Show a leftmost derivation, rightmost derivation, and derivation tree for abaabb.

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Leftmost derivation:

$$S \Rightarrow SA \Rightarrow AA \Rightarrow aAbA \Rightarrow abA \Rightarrow abaAb \Rightarrow abaaAbb \Rightarrow abaabb$$

Rightmost derivation:

$$S \Rightarrow SA \Rightarrow SaAb \Rightarrow SaaAbb \Rightarrow Saabb \Rightarrow Aaabb \Rightarrow aAbaabb \Rightarrow abaabb$$

Derivation tree:

