Automata Theory: Solutions 6

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Problem 1
Demonstrate that, if L_1 is a regular language on the alphabet $\Sigma = \{a, b\}$, then the following subset of L_1 is also a regular language:

$$L_2 = \{w : \ w \in L_1 \text{ and } w \text{ includes at least one } b\}.$$

Consider the language L_3 defined by the expression a^*; since we describe L_3 by a regular expression, it is a regular language. This language includes all strings that have no b, and we can express L_2 as follows:

$$L_2 = L_1 - L_3.$$

Thus, L_2 is the difference of two regular languages, which implies that it is also regular.

Problem 2
Argue that the language $\{a^n b^{2n} : \ n \geq 0\}$ is not regular.

We suppose that the language is regular and derive a contradiction. If this language is regular, there is some m-state DFA that accepts it, where m is an unknown fixed number.

Suppose that we begin from the initial state of the DFA and trace the path for each of the following strings: $a^0, a^1, a^2, ..., a^m$. Since the total number of these strings is $m + 1$, two of them must lead to the same state; we denote these two strings a^i and a^j (see the picture).

Since the automaton accepts the string $a^i b^{2i}$, this string must lead from the initial state to a final state, as shown in the picture. Then, the string $a^j b^{2i}$ leads to the same final state, contradicting the fact that $a^j b^{2i}$ is *not* in the language.