Automata Theory: Solutions 3

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                                    X
                                    X   X   X
                            X       X   X   X       X
number of                   X       X   X   X       X
homeworks                   X       X   X   X   X   X
                            X   X   X   X   X   X   X
                X           X   X   X   X   X   X   X
                ----------------------------
                1   2   3   4   5   6   7   8   9   10
                              grades
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The histogram shows the distribution of grades, from 0 to 10.

Problem 1
Consider the following two languages on the alphabet $\Sigma = \{a, b\}$:

$L_1 = \{a^n : \ n \geq 1\}$
$L_2 = \{b^n : \ n \geq 1\}$

Describe the following languages:

- $L_3 = L_1^* = {L_1}^0 \cup {L_1}^1 \cup {L_1}^2 \cup \ldots = \{a^n : \ n \geq 0\}$
  (all strings that have no b's)
- $L_4 = \overline{L_1} = \{\lambda\} \cup \{w : \ w \text{ includes at least one } b\}$
  (the empty string and all strings that have some b's)
- $L_5 = L_1 \cup L_2 = \{a^n, b^n : \ n \geq 1\}$
  (all strings that have only a's or only b's)
- $L_6 = L_1 L_2 = \{a^m b^n : \ m \geq 1, \ n \geq 1\}$
- $L_7 = ({L_1}^2)({L_2}^2)({L_1}^2) = \{a^m b^n a^k : \ m, n, k \geq 2\}$
- $L_8 = (L_1 \cup L_2)^* = \Sigma^*$
  (all strings on the alphabet)
- $L_9 = (L_1 L_2)^* = \{awb : \ w \text{ is any string on } \Sigma\}$
  (all strings that begin with a and end with b)

Problem 2
Consider the alphabet $\Sigma = \{a, b\}$. Is there any language L on this alphabet for which $(\overline{L})^* = \overline{L^*}$? If yes, give an example of such a language; if no, explain why.

By definition, the star-closure of any language includes the empty string λ. In particular, the $(\overline{L})^*$ language, which is the star-closure of $\overline{L}$, includes λ. On the other hand, since L^* includes λ, we conclude that its complement, $\overline{L^*}$, does *not* include λ. Thus, the empty string is included into $(\overline{L})^*$, and it is not included into $\overline{L^*}$, which implies that $(\overline{L})^* \neq \overline{L^*}$.