Automata Theory: Assignment 1

Due date: August 30 (Thursday)

Problem 1 (4 points)
Consider the following sets of integer numbers:

$S_1 = \{4, 5, 6\}$
$S_2 = \{i : \ i \text{ is even}\}$
$S_3 = \{i : \ i \text{ is divisible by } 3\}$

For each set below, specify its elements and determine whether it is finite or infinite:

$S_4 = S_1 \times S_1$
$S_5 = 2^{S_1}$
$S_6 = S_1 \cap S_2$
$S_7 = S_2 \cap S_3$

Problem 2 (2 points)
Prove that, if $S_1 \subseteq S_2$, then $\overline{S_2} \subseteq \overline{S_1}$.

Problem 3 (4 points)
Prove the following equalities:

(a) $1 + 2 + 3 + 4 + ... + n = \frac{n \cdot (n+1)}{2}$.

(b) $1 + x + x^2 + x^3 + ... + x^n = \frac{x^{n+1} - 1}{x - 1}$ (where $x \neq 1$).