Analysis of Algorithms: Solutions 1

```
                                   X
                                X  X
                             X  X  X  X
                             X  X  X  X
number of                    X  X  X  X  X
homeworks                    X  X  X  X  X
                             X  X  X  X  X
           X  X        X  X  X  X  X  X  X
           X  X     X  X  X  X  X  X  X  X
           --------------------------------
           0  1  2  3  4  5  6  7  8  9  10
                        grades
```

This histogram shows the distribution of grades (from 0 to 10) for the homeworks submitted on time; it does not include the late submissions. The mean value of these grades is 6.82.

Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an *inversion*. For example, the array $\langle 2, 3, 8, 6, 1 \rangle$ contains five inversions.

Problem 1
What array with elements from the set $\{1, 2, ..., n\}$ has the most inversions? How many inversions does it have?

The array $\langle n, n-1, ..., 2, 1 \rangle$ has the most inversions. Every pair of elements in this array is an inversion. Since the total number of pairs is $\frac{n(n-1)}{2}$, the total number of inversions in this array is also $\frac{n(n-1)}{2}$.

Problems 2 and 3
(2) Give an algorithm that inputs an array and outputs the number of inversions in the array. You may assume that all elements of the array are distinct.
(3) Estimate the worst-case running time of your algorithm.

INVERSIONS(A, n)	*cost*	*times*
counter $\leftarrow 0$	c_1	1
for $i \leftarrow 1$ **to** $n-1$	c_2	n
do for $j \leftarrow i+1$ **to** n	c_3	$\sum_{i=1}^{n-1}(n-i+1) = \frac{n(n+1)}{2} - 1$
do if $A[i] > A[j]$	c_4	$\sum_{i=1}^{n-1}(n-i) = \frac{n(n-1)}{2}$
then *counter* $\leftarrow$ *counter* $+1$	c_5	$\leq \sum_{i=1}^{n-1}(n-i) = \frac{n(n-1)}{2}$
return *counter*	c_6	1

$$
\begin{aligned}
T(n) &\leq c_1 + c_2 n + c_3\left(\frac{n(n+1)}{2} - 1\right) + c_4 \frac{n(n-1)}{2} + c_5 \frac{n(n-1)}{2} + c_6 && (1)\\
&= \left(\frac{c_3}{2} + \frac{c_4}{2} + \frac{c_5}{2}\right)n^2 + \left(c_2 + \frac{c_3}{2} - \frac{c_4}{2} - \frac{c_5}{2}\right)n + (c_1 - c_3 + c_6) && (2)\\
&= \Theta(n^2) && (3)
\end{aligned}
$$