Algorithms (COT 6405): Solutions 8

Problem 1

Give a nonrecursive algorithm that prints all elements of a binary search tree in sorted order.

Iterative-Tree-Walk(T)
$x \leftarrow$ Tree-Minimum($root[T]$)
while $x \neq$ NIL
  do print $key[x]$
    $x \leftarrow$ Tree-Successor(x)

The running time is $\Theta(n)$, where n is the number of nodes in the tree.

Problem 2

Suppose that we have four algorithms, called A_0, A_1, A_2, and A_3, whose respective running times are n, n^2, $\lg n$, and 2^n. If we use a certain old computer, then the maximal sizes of problems solvable in one hour by these algorithms are s_0, s_1, s_2, and s_3. Suppose that we have replaced the old computer with a new one, which is k times faster. Now the maximal size of problems solvable in one hour by A_0 is $k \cdot s_0$. What are the maximal problem sizes for the other three algorithms if we run them on the new computer?

For A_1**:** On the old computer, A_1 solves a problem of size s_1 in one hour. The running time of this algorithm on a problem of size s_1 is s_1^2; hence, $s_1^2 = 1$ hour. The new computer is k times faster, which means that the running time of A_1 is n^2/k. We denote the size of the largest problem solvable in one hour on the new computer by v_1; then, $v_1^2/k = 1$ hour. We conclude that $v_1^2/k = s_1^2$, which implies that $v_1 = s_1 \cdot \sqrt{k}$. Thus, the maximal size of a problem solvable in one hour on the new computer is $s_1 \cdot \sqrt{k}$.

For A_2**:** On the old computer, A_2 solves a problem of size s_2 in one hour, which means that $\lg s_2 = 1$ hour. If we denote the maximal size of a problem solvable in one hour on the new computer by v_2, then $\lg v_2/k = 1$ hour. We conclude that $\lg v_2/k = \lg s_2$, which implies that $v_2 = s_2^k$. Thus, the maximal problem solvable in one hour on the new computer is of size s_2^k.

For A_3**:** We denote the maximal problem solvable by A_3 on the new computer by v_3, and use a similar reasoning to obtain the equation $2^{v_3}/k = 2^{s_3}$, which implies that $v_3 = s_3 + \lg k$.